

# Mathematics Instructional Practices And Learner Reasoning Facilitation Of Key Stage 2 Teachers In Ambaguio, Nueva Vizcaya

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## Publication history:

Received: September 2, 2026

Revised: September 28, 2026

Accepted: October 1, 2026

Published: October 2, 2026

[International Journal of Sustainable and Integrated Studies, Vol. 1, No. 3, pp. 10–21, October 2026.](#)

## Abstract

This study examined mathematics instructional practices and learner reasoning facilitation among Key Stage 2 teachers in Ambaguio, Nueva Vizcaya and developed a Reasoning-Centered Mathematics Instruction Model based on the findings. Anchored in Social Constructivist Theory, Bruner's Theory of Representation, Cognitive Load Theory, and the Mathematical Proficiency Framework, the study employed a predictive-correlational developmental research design. Sixty-eight Key Stage 2 Mathematics teachers from public elementary schools were selected through simple random sampling and surveyed using a researcher-made, expert-validated questionnaire with high internal consistency. Weighted mean, standard deviation, Pearson product-moment correlation, and hierarchical multiple regression were used for data analysis. Mathematics instructional practices were implemented Often ( $M = 3.88$ ), with contextualization of mathematical learning obtaining the highest mean ( $M = 4.03$ ), while learner reasoning facilitation was likewise rated Often ( $M = 3.74$ ), with transfer to real-life problem situations highest ( $M = 3.86$ ). Mathematics instructional practices were strongly, positively, and significantly related to learner reasoning facilitation ( $r = .74$ ,  $p < .001$ ). The instructional-practice dimensions collectively explained 61% of the variance in learner reasoning facilitation, with mathematical discourse facilitation ( $\beta = .32$ ,  $p = .003$ ), problem-based task structuring ( $\beta = .25$ ,  $p = .011$ ), and concept development and representation ( $\beta = .18$ ,  $p = .047$ ) emerging as significant predictors. The findings support strengthening problem-based tasks, mathematical discourse, strategy comparison, pattern recognition, generalization, and learner-generated problems while sustaining established instructional strengths. The resulting model follows the recurring process Represent  $\rightarrow$  Challenge  $\rightarrow$  Discuss  $\rightarrow$  Analyze  $\rightarrow$  Transfer. The study aligns primarily with SDG 4—Quality Education and secondarily with SDG 10—Reduced Inequalities through its emphasis on teaching quality, reasoning, and equitable participation. Its sustainability contribution is primarily educational and institutional by supporting durable mathematical understanding, teacher capacity, professional development, and sustained instructional improvement.

**Keywords:** Conceptual Understanding, Contextualization of Mathematical Learning, Key Stage 2, Learner Reasoning Facilitation, Mathematical Discourse Facilitation, Mathematics Instructional Practices, Problem-Based Task Structuring, Reasoning-Centered Mathematics Instruction Model, Sustainable Education

## 1. Introduction

Mathematics education increasingly emphasizes reasoning, problem solving, communication, and conceptual understanding rather than the memorization of procedures. At the elementary level, learners need opportunities to connect new knowledge with prior understanding, explain processes, recognize relationships, and apply mathematical ideas in unfamiliar situations. Recent research on school mathematics reasoning emphasizes opportunities for learners to construct and substantiate mathematical claims across concept introduction, procedures, exercises, and problem-solving situations (Valenta et al., 2024). This orientation is especially important in Key Stage 2, when learners move beyond basic numeracy toward more complex mathematical ideas and more abstract forms of thinking. The present study examines mathematics instructional practices and learner reasoning facilitation among Key Stage 2 teachers in Ambaguio, Nueva Vizcaya. It focuses on concept development and representation, problem-based task structuring, mathematical discourse facilitation, formative feedback and error analysis, and contextualization of mathematical learning, and relates these practices to explanation and justification, strategy comparison and selection, pattern recognition and generalization, reasoning through mathematical errors, and transfer to real-life problem situations. The study is intended to provide a basis for a Reasoning-Centered Instruction Model suited to the instructional realities of Key Stage 2 mathematics.

Mathematics instructional practices encompass the approaches, routines, questions, tasks, assessments, representations, and teacher–learner interactions used to support mathematical learning. Effective instruction gives learners opportunities to construct meaning, test ideas, discuss strategies, learn from errors, and apply mathematics in meaningful situations. Recent studies show that multiple representations are most useful when teachers explicitly connect their mathematical meanings and help learners coordinate ideas across forms (Cordero-Siy & Ghousseini, 2022; Post & Prediger, 2024). Learner reasoning facilitation complements these practices by helping pupils develop, express, and refine mathematical thinking through probing questions, multiple solution paths, connected representations, misconception analysis, and justification. Research on mathematical discourse likewise emphasizes learning communities in which students articulate reasoning and make sense of peers' contributions (Hayden et al., 2024; Schmidt et al., 2024). For Grades 4 to 6, the instructional challenge is to balance procedural fluency with conceptual understanding. Learners work with larger numbers, fractions, decimals, measurement, geometry, and data, but successful performance depends on whether instruction enables them to understand relationships and choose and justify appropriate strategies.

The literature consistently presents concept development and representation as foundations of meaningful mathematics learning. Representations such as manipulatives, diagrams, number lines, tables, graphs, drawings, and symbols can make abstract ideas visible, but their value depends on how learners and teachers connect the underlying mathematical meanings (Cordero-Siy & Ghousseini, 2022; Cartwright, 2024; Post & Prediger, 2024). Student-generated representations can also support generalization and early algebraic reasoning (Schifter & Russell, 2022). Problem-based task structuring creates opportunities for learners to analyze conditions, select strategies, test ideas, and justify conclusions. Contemporary reviews of productive struggle emphasize challenging tasks with multiple entry points and solutions, together with teacher support that preserves students' opportunities to reason (Young et al., 2024; Biccard, 2024; Baker et al., 2024). Mathematical discourse makes learners' thinking available for examination. Teachers strengthen collective understanding when they elicit ideas, press for explanation, and create opportunities for learners to respond to one another (Hayden et al., 2024; Schmidt et al., 2024; Ozturk, 2025). Formative feedback and error analysis are similarly central because they use evidence of learner thinking to guide improvement. A recent systematic review identifies feedback, diagnostics, and adaptive instructional action as core components of formative assessment in mathematics (Maskos et al., 2025). Errors can become opportunities for deeper reasoning when teachers discuss and explain them rather than merely correct them (Kaufmann et al., 2023; Benecke & Kaiser, 2023; Janíková et al., 2024). Context-responsive instruction is also important where language, culture, and lived experience influence participation in mathematical activity; recent work links equitable mathematics participation with relational noticing, multilingual discourse, and culturally responsive instructional design (Mendez & van Es, 2025; Ingram et al., 2024; Jiménez-Silva et al., 2025). The literature on learner reasoning further emphasizes explanation, justification, conjecturing, generalization, and the construction and substantiation of mathematical claims (Valenta et al., 2024; Weingarden & Buchbinder, 2023). Teachers' professional noticing is important because responsive instruction depends on interpreting learner thinking and deciding how to act on it; recent reviews and elementary studies confirm the continuing centrality of this competence (Weyers et al., 2024; Jacobs et al., 2024; Jessup, 2025). Across these themes, instructional quality and reasoning facilitation are interdependent: concept development, representations, rich tasks, discourse, feedback, error analysis, and contextualization create the conditions in which learners can explain, compare, generalize, and transfer ideas.

Globally, mathematics education has moved toward instruction that develops conceptual understanding, reasoning, problem solving, discourse, and application. International evidence also shows that learners' ability to use mathematics in complex and real-life situations remains a central educational concern. The Programme for International Student Assessment 2022 identified continuing challenges in mathematical reasoning and

application, including for Filipino learners (Organisation for Economic Co-operation and Development, 2023). In the Philippines, the need to strengthen mathematics learning is reflected in curriculum reform. The MATATAG Mathematics Curriculum identifies Key Stage 2 as Grades 4 to 6 and expects learners to extend understanding across numbers, algebra, measurement, geometry, data, and probability while using written and mental methods across contexts (Department of Education [DepEd], 2023). The Philippines' below-OECD-average performance in PISA 2022 further underscores difficulties in applying mathematics to real-life and complex situations (Organisation for Economic Co-operation and Development [OECD], 2023). These conditions strengthen the need for classroom practices that develop analysis, justification, problem solving, and understanding rather than routine computation alone. The local context of Ambaguio, Nueva Vizcaya adds instructional realities related to learner diversity, resources, community background, curriculum expectations, and school support. Key Stage 2 teachers in geographically and culturally distinct settings may work with varying readiness levels, limited learning materials, small-school or multigrade conditions in some areas, and a need to connect lessons to learners' lived experiences. The study therefore examines how reasoning-centered mathematics instruction is enacted within this specific context rather than assuming that practices documented elsewhere operate identically in Ambaguio.

The study is anchored in Social Constructivist Theory, which views learning as active and socially mediated through interaction, language, guidance, and meaningful participation (Vygotsky, 1978). In mathematics, learners develop reasoning through discussion, explanation, questioning, collaboration, and guided engagement. This perspective supports instructional practices involving discourse, scaffolding, questioning, and collaborative problem solving. Bruner's Theory of Representation supports the use of enactive, iconic, and symbolic forms so that learners can connect concrete experience, visual models, and abstract notation (Bruner, 1966). This is particularly relevant to Key Stage 2 learners as they transition toward more complex mathematical ideas. Cognitive Load Theory complements this view by emphasizing instructional designs that manage working-memory demands so that unnecessary mental burden does not interfere with learning (Sweller, 1988). The Mathematical Proficiency Framework further positions adaptive reasoning alongside conceptual understanding, procedural fluency, strategic competence, and productive disposition (Kilpatrick et al., 2001). These older sources are retained as original theories and foundational frameworks rather than as current empirical evidence. Conceptually, mathematics instructional practices constitute the study's independent variable through five dimensions: concept development and representation, problem-based task structuring, mathematical discourse facilitation, formative feedback and error analysis, and contextualization of mathematical learning. Learner reasoning facilitation constitutes the dependent variable through explanation and justification of solutions, strategy comparison and selection, pattern recognition and generalization, reasoning through mathematical errors, and transfer of reasoning to real-life problem situations. The study examines the relationship between these variables, identifies instructional dimensions that predict reasoning facilitation, and uses the findings as the basis for a Reasoning-Centered Instruction Model. This narrative replaces the original research-paradigm figure while retaining its conceptual relationships. Instructional design also provides a practical bridge between theory and classroom implementation. The study retains Richey and Klein (2016) as an established methodological source for design and development research. In this study, coherence is expressed through the expectation that representations, tasks, discourse, feedback, error analysis, and contextualization should operate together to create sustained opportunities for mathematical reasoning.

Recent studies reinforce the importance of reasoning-centered instruction while also showing persistent implementation gaps. Colonnese et al. (2026) demonstrated the value of teacher prompting and opportunities for written justification in making learner reasoning visible. Smith and Ellis (2026) highlighted the influence of teachers' perceptions of learners' mathematical capabilities on opportunities for explanation and critique. Huang et al. (2026) showed that technology-supported formative assessment depends on teachers' knowledge, judgment, and capacity to interpret learner reasoning rather than on technology alone. Johnson and Roy (2026) emphasized multiple representations, conceptual coherence, attention to student thinking, generalization, and error analysis as important features of conceptual mathematics instruction, while Wardat et al. (2026) further situated reasoning-oriented practice within contemporary mathematics teaching conditions. Studies also show that discourse and reasoning require deliberate facilitation. Ozturk (2025) found that eliciting ideas, introducing key concepts, and encouraging learners to take positions were consequential in mathematical modeling discourse. Livy et al. (2025) showed how carefully sequenced questioning and student work samples can support interpretation, misconception analysis, and strategy comparison. Jiménez-Silva et al. (2025) integrated attention to language, mathematical thinking, and cultural responsiveness in supporting multilingual mathematical discourse, while Egara and Mosia (2025) connected reasoning-oriented instruction with culturally relevant pedagogy, differentiated instruction, and universal design for learning. Teacher learning and professional support emerge as another gap. Ingram et al. (2024) identified language-responsive practices needed to sustain mathematical meaning in linguistically complex settings. Ebby et al. (2024) showed that focused facilitation of professional discourse can strengthen teachers' capacity to study instructional artifacts and learner thinking. Schmidt et al. (2024) found that effective discourse practices can develop through intentional effort, learning experiences, professional relationships, and pedagogical knowledge. Bogota (2024) demonstrated how translanguaging can support conceptual explanation, participation, and confidence among language-minoritized learners. At the same time, ambitious instructional practices are not

consistently enacted. Auslander et al. (2023) found substantial variability in learner-centered and equitable mathematics teaching even among experienced teachers, with practice shaped by specialized knowledge and contextual constraints. Bozkuş and Guner (2023) showed that frequent teacher questioning does not necessarily produce productive reasoning when classroom moves quickly close down learner ideas. Çelebi (2023) found that teacher noticing may remain interpretive without being translated into actionable instructional decisions. Muir (2022) similarly showed that productive mathematical discourse in online settings depends on purposeful social, cognitive, and teaching presence rather than occurring automatically. Professional development studies likewise indicate that improving reasoning facilitation requires more than introducing materials or content. Maboya et al. (2022) found that sustained professional development helped primary mathematics teachers reconsider learner errors, strengthen content knowledge, and improve pedagogical use of manipulatives. Weyers et al. (2024) synthesized recent teacher-noticing research and emphasized the importance of teachers' professional interpretation of classroom events, while Mendez and van Es (2025) connected relational noticing with equitable mathematics participation. Together, these studies establish a clear rationale for the present investigation. Existing research demonstrates the value of concept-focused instruction, representation, discourse, formative assessment, culturally responsive practice, professional noticing, and explicit reasoning support, yet it also documents uneven enactment and context-dependent constraints. What remains insufficiently addressed in the reviewed literature is the combined examination of the specified mathematics instructional-practice dimensions and learner-reasoning dimensions among Key Stage 2 teachers in Ambaguio, Nueva Vizcaya, including which instructional dimensions significantly predict reasoning facilitation. The study addresses this contextual and integrative gap and uses the findings to develop a reasoning-centered mathematics instruction model suited to the setting.

The study aligns most directly with SDG 4 – Quality Education because it focuses on the quality of mathematics teaching and on instructional practices that develop understanding, reasoning, problem solving, communication, and meaningful application. Its emphasis on equitable participation and reasoning-centered mathematics is consistent with recent research on relational noticing, multilingual mathematical discourse, and equitable participation (Mendez & van Es, 2025; Jiménez-Silva et al., 2025). By examining how Key Stage 2 teachers facilitate reasoning and by developing an instructional model from the findings, the study supports efforts to strengthen teaching quality, foundational learning, and teacher capacity within basic education. A secondary connection exists with SDG 10 – Reduced Inequalities insofar as the reviewed literature emphasizes equitable participation, culturally and linguistically responsive instruction, differentiated access, and the use of learners' varied resources in mathematical discourse. These concerns are particularly relevant where learners differ in readiness, language, culture, and access to instructional resources. The study does not measure inequality directly; rather, its contribution lies in examining instructional practices that can make mathematical reasoning more accessible within diverse classroom conditions.

The study contributes primarily to educational sustainability by examining teaching practices that can support durable mathematical understanding rather than short-term procedural performance. Reasoning-centered instruction develops learners' capacity to explain, justify, compare, generalize, analyze errors, and apply mathematics across contexts, abilities that support continued learning as mathematical demands become more complex. Its attention to teacher practice also supports institutional sustainability because the findings may inform instructional supervision, coaching, learning action cells, mentoring, curriculum implementation, and professional development within schools. The study also has socio-economic and community sustainability relevance because mathematical reasoning supports practical decision-making and the interpretation of quantities, measurements, data, and everyday problems. Recent work on culturally and linguistically responsive mathematics teaching emphasizes connecting mathematical participation and discourse with learners' linguistic and relational resources (Ingram et al., 2024; Mendez & van Es, 2025; Jiménez-Silva et al., 2025). These contributions remain educational in scope: the study does not directly measure economic, environmental, or community sustainability outcomes, but it examines classroom capacities that can support adaptable, informed, and inclusive learning. Finally, the proposed Reasoning-Centered Instruction Model may contribute to the sustainability of instructional improvement by organizing concept development, representation, problem-based tasks, discourse, formative feedback, error analysis, and contextualization around the common goal of learner reasoning.

## **2. Material and methods**

### **2.1 Research Design**

The study employed a predictive-correlational developmental research design. The correlational component determined the relationship between mathematics instructional practices and learner reasoning facilitation, while the predictive component identified which dimensions of mathematics instructional practices significantly predicted learner reasoning facilitation. The developmental component used the findings as the basis for developing a reasoning-centered mathematics instruction model. This design enabled the study to describe the variables, examine their relationship, determine the predictive contribution of specific instructional practices, and develop an instructional model grounded in the results.

## **2.2 Locale of the Study**

The study was conducted in Ambaguio, Nueva Vizcaya, particularly in public elementary schools offering Key Stage 2. The locale provided an appropriate setting for examining mathematics instruction under actual classroom conditions. It allowed the study to investigate how teachers implemented mathematics instructional practices and facilitated learner reasoning, including explanation, strategy use, error analysis, and the application of mathematics to real-life situations.

## **2.3 Respondents of the Study**

The respondents were 68 Key Stage 2 teachers handling Mathematics in public elementary schools in Ambaguio, Nueva Vizcaya. They were included because they directly taught Mathematics to learners in Grades 4 to 6, where reasoning, problem solving, representation, and conceptual understanding are increasingly developed. The study focused specifically on their mathematics instructional practices and learner reasoning facilitation and did not include respondent profile variables, learner achievement scores, classroom observation ratings, or the perspectives of parents and learners. The delimitations of the original study likewise specify Key Stage 2 teachers handling mathematics in Ambaguio, Nueva Vizcaya.

## **2.4 Sample and Sampling Procedure**

The study used simple random sampling to select 68 Key Stage 2 Mathematics teachers. A complete list of qualified teachers was obtained from the appropriate office, after which the respondents were randomly selected. This procedure gave each qualified teacher an equal opportunity to participate and helped minimize selection bias.

## **2.5 Research Instrument**

A researcher-made survey questionnaire was used to measure mathematics instructional practices and learner reasoning facilitation. The mathematics instructional practices component covered concept development and representation, problem-based task structuring, mathematical discourse facilitation, formative feedback and error analysis, and contextualization of mathematical learning. These dimensions correspond to the instructional practices defined in the study. The learner reasoning facilitation component measured explanation and justification of solutions, strategy comparison and selection, pattern recognition and generalization, reasoning through mathematical errors, and transfer of reasoning to real-life problem situations. Learner reasoning facilitation was operationally defined as teachers' practices that help learners explain, justify, compare, generalize, analyze errors, and apply mathematical reasoning in solving problems. The questionnaire used a five-point Likert scale in which 5 represented Always, 4 represented Often, 3 represented Sometimes, 2 represented Rarely, and 1 represented Never. This scale measured how frequently teachers implemented the practices described in the instrument. The instrument underwent content validation by experts in mathematics education, educational research, curriculum implementation, and educational leadership. The validators assessed the clarity, relevance, organization, appropriateness, and alignment of the items with the intended constructs and research questions. Their recommendations were incorporated into the revised instrument. Content validation yielded an S-CVI/Ave of 0.96, while the retained items obtained Item-Level Content Validity Index values ranging from 0.83 to 1.00, indicating excellent content validity. The revised questionnaire was pilot tested among teachers with comparable teaching responsibilities who were not included in the actual study. Reliability was established using Cronbach's alpha. For Mathematics Instructional Practices, the coefficients were 0.92 for concept development and representation, 0.91 for problem-based task structuring, 0.93 for mathematical discourse facilitation, 0.92 for formative feedback and error analysis, and 0.90 for contextualization of mathematical learning, with an overall coefficient of 0.95. For Learner Reasoning Facilitation, the coefficients were 0.92 for explanation and justification of solutions, 0.90 for strategy comparison and selection, 0.91 for pattern recognition and generalization, 0.93 for reasoning through mathematical errors, and 0.89 for transfer of reasoning to real-life problem situations, with an overall coefficient of 0.94. The entire questionnaire obtained a Cronbach's alpha of 0.96, establishing adequate content validity and high internal consistency.

## **2.6 Data Gathering Procedure**

The researcher first obtained approval from the concerned school and division authorities and coordinated with school heads regarding the schedule and administration of the questionnaire. Before data collection, the respondents were informed of the study's purpose, the voluntary nature of participation, the confidentiality of their responses, and their right to withdraw without penalty. The validated questionnaire was then administered to the identified Key Stage 2 Mathematics teachers with clear instructions for completion. After administration, the accomplished questionnaires were retrieved and checked for completeness and consistency. Valid responses were encoded, organized, and prepared for statistical analysis, and the collected data were treated objectively and used solely for the purposes of the study.

## **2.7 Data Analysis Techniques**

Weighted mean and standard deviation were used to determine the extent of mathematics instructional practices and the level of learner reasoning facilitation. The weighted mean represented the respondents' general ratings, while the standard deviation indicated the consistency or variability of their responses. The Pearson product-

moment correlation coefficient was used to determine the strength, direction, and significance of the relationship between mathematics instructional practices and learner reasoning facilitation based on their composite scores. Hierarchical multiple regression analysis was employed to determine which dimensions of mathematics instructional practices significantly predicted learner reasoning facilitation. The analysis examined the relative contributions of concept development and representation, problem-based task structuring, mathematical discourse facilitation, formative feedback and error analysis, and contextualization of mathematical learning. Changes in explained variance, standardized regression coefficients, significance values, and overall model fit were considered in determining the explanatory contributions of the predictors. The results of the descriptive, correlational, and regression analyses served as the basis for developing the Reasoning-Centered Mathematics Instruction Model. All inferential analyses were evaluated at the 0.05 level of significance. A p-value below 0.05 was considered statistically significant and resulted in rejection of the corresponding null hypothesis, whereas a p-value equal to or greater than 0.05 indicated insufficient evidence to reject the null hypothesis.

## **2.8 Ethical Considerations**

Ethical standards were observed throughout the study to protect the rights, dignity, privacy, and welfare of the respondents. Permission was obtained from the concerned division and school authorities before data collection. Respondents were informed about the purpose of the research, the voluntary nature of their participation, and their right to decline or withdraw without penalty. Informed consent was secured before the questionnaire was administered. Confidentiality and anonymity were maintained by preventing the disclosure of individual identities in the analysis, interpretation, and reporting of the findings. The collected information was used solely for academic and research purposes and was handled securely and objectively. The researcher ensured that participation posed no unnecessary physical, psychological, professional, or social risk. Data collection, storage, analysis, and reporting were conducted with integrity, without fabrication, falsification, alteration, or manipulation of information.

## **3. Results and Discussion**

### **3.1 Mathematics Instructional Practices**

Overall, mathematics instructional practices obtained a weighted mean of 3.88, interpreted as Often, indicating that teachers regularly employed practices supporting conceptual understanding, problem solving, mathematical communication, formative assessment, and meaningful application. Contextualization of mathematical learning received the highest mean of 4.03, followed by concept development and representation at 3.93, formative feedback and error analysis at 3.89, mathematical discourse facilitation at 3.81, and problem-based task structuring at 3.73. The overall standard deviation of 0.48 indicated reasonable consistency in teachers' responses. The findings show that teachers were particularly consistent in making mathematics meaningful, concrete, and understandable through familiar contexts, representations, and formative support. Practices demanding greater learner autonomy, particularly extended problem solving and discourse, were comparatively less frequent because these require teachers to allow multiple approaches, provide sufficient thinking time, and manage varied learner responses. The reviewed literature supports the integration of concept development, representation, problem-based tasks, discourse, feedback, error analysis, and contextualized learning. Auslander et al. (2023) likewise reported considerable variability even among experienced mathematics teachers and cautioned that learner-centered reasoning practices are affected by specialized knowledge and classroom constraints, demonstrating why instructional dimensions may not be enacted with equal frequency.

#### **3.1.1 Concept Development and Representation**

Concept development and representation obtained an overall weighted mean of 3.93 and a standard deviation of 0.52, interpreted as Often. Checking learners' conceptual understanding before procedural practice received the highest mean of 4.11, followed by connecting mathematics to prior knowledge at 4.08 and providing varied examples at 4.06. Comparatively lower means were obtained for guiding learners to connect concrete examples to mathematical symbols at 3.79 and asking learners to explain mathematical ideas in their own words at 3.72. These results indicate that teachers regularly established conceptual understanding but provided somewhat fewer opportunities for learners to independently verbalize concepts and connect concrete experiences with symbolic representations. The findings indicate that conceptual instruction was generally established before procedural work, allowing teachers to identify misconceptions and provide multiple entry points into mathematical ideas. However, greater emphasis on learner-generated explanations and explicit transitions among concrete, visual, verbal, and symbolic forms could further deepen understanding. The result is corroborated by the related literature indicating that concept development becomes deeper when new mathematical knowledge is connected to prior understanding and when teachers help pupils move across concrete, visual, verbal, and symbolic representations. Johnson and Roy (2026) likewise emphasized the deliberate use of multiple representations and attention to student thinking as essential components of conceptual mathematics instruction, particularly when learners require carefully mediated opportunities to reason.

### **3.1.2 Problem-Based Task Structuring**

Problem-based task structuring obtained an overall mean of 3.73, interpreted as Often. Providing challenging but manageable mathematics activities received the highest mean of 3.89, followed by tasks requiring learners to explain how they solved problems at 3.83 and follow-up questioning after problem-solving activities at 3.82. Lower means were obtained for independent problem solving before whole-class discussion at 3.59 and providing sufficient time for learners to explore possible solutions at 3.56. Thus, teachers regularly provided cognitively demanding tasks but were less consistent in allowing prolonged independent exploration. The findings suggest that teachers generally provided appropriate mathematical challenges and encouraged reasoning beyond obtaining correct answers. However, limited exploration time and independent problem solving may reduce opportunities for productive struggle and strategy development. The finding is supported by Young et al. (2024) and Baker et al. (2024), who emphasize that productive struggle depends on challenging tasks, sustained reasoning, and teacher support that does not remove learners' opportunities to think. The related literature further explains that the reasoning potential of a mathematical task depends not merely on task selection but on how the teacher introduces, supports, discusses, and concludes it. This corroborates the comparatively lower ratings for exploration time and independent solution development.

### **3.1.3 Mathematical Discourse Facilitation**

Mathematical discourse facilitation obtained an overall mean of 3.81, interpreted as Often. Guiding learners to listen respectfully to different mathematical ideas received the highest mean of 4.03, followed by pair or group discussion at 3.93 and summarizing learners' ideas and connecting them to lesson concepts at 3.90. Asking learners to explain their thinking and using probing questions obtained means of 3.88 and 3.86, respectively. Lower means were recorded for helping learners use appropriate mathematical terms at 3.65 and allowing learners to ask questions about classmates' solutions at 3.53. The results indicate that mathematical discussion was regularly incorporated into instruction, although discourse remained more strongly teacher-facilitated than learner-to-learner. Strengthening opportunities for pupils to question, clarify, challenge, and extend their peers' mathematical reasoning could promote more sustained mathematical interaction. The findings are corroborated by Colonnese et al. (2026), who reported that learner reasoning becomes more visible when teachers deliberately prompt explanation and create structured opportunities for justification. Schmidt et al. (2024) also demonstrated that productive mathematics discourse is a teachable pedagogical practice that grows through repeated opportunities to plan and lead discussions where learners articulate reasoning and respond to peers.

### **3.1.4 Formative Feedback and Error Analysis**

Formative feedback and error analysis obtained an overall mean of 3.89, interpreted as Often. Encouraging learners to learn from mistakes without embarrassment received the highest mean of 4.13, followed by providing corrective feedback during practice activities at 4.07. Using formative assessment results to adjust mathematics instruction obtained 3.97, while giving feedback intended to improve mathematical thinking obtained 3.94. Lower means were recorded for asking learners to identify where their solution process went wrong at 3.72 and explain how they corrected mistakes at 3.68. These results indicate that teachers generally maintained a supportive approach to errors and regularly used learner performance to guide subsequent instruction. However, error analysis remained comparatively teacher-directed, with learners less frequently required to reconstruct and explain their own erroneous reasoning. The finding is consistent with the reviewed literature, which explains that formative feedback should help learners recognize what they understand, what requires improvement, and what next step will strengthen reasoning. The same literature treats errors as windows into mathematical thinking rather than merely failures. Huang et al. (2026) similarly emphasized that formative assessment contributes to reasoning only when teachers interpret misconceptions and use evidence to adapt questions, representations, and scaffolds.

### **3.1.5 Contextualization of Mathematical Learning**

Contextualization of mathematical learning obtained an overall mean of 4.03, the highest among the instructional-practice dimensions. Helping learners recognize the importance of mathematics outside the classroom received the highest mean of 4.20, followed by relating lessons to familiar real-life situations at 4.18 and demonstrating the practical use of mathematics at 4.17. Using daily experiences, everyday problems, and familiar objects also received high ratings of 4.14, 4.10, and 4.08, respectively. In contrast, encouraging learners to create problems based on their own real-life experiences obtained 3.36, interpreted as Sometimes. The results demonstrate that teachers frequently connected mathematics with practical and familiar situations, making abstract ideas more relevant to learners. However, contextualization was predominantly teacher-generated because learners were less frequently asked to construct mathematical problems from their own experiences. The result agrees with the related literature describing real-life mathematical reasoning as the application of mathematics to time, distance, prices, measurements, schedules, resources, and data, requiring interpretation and judgment in addition to computation. The reviewed studies further emphasize culturally responsive and context-sensitive mathematics instruction, particularly where meaningful participation depends on connecting mathematical language and concepts with learners' lived experiences.

### **3.2 Learner Reasoning Facilitation**

Learner reasoning facilitation obtained an overall weighted mean of 3.74, interpreted as Often. Transfer of reasoning to real-life problem situations ranked highest at 3.86, followed by explanation and justification of solutions at 3.81, reasoning through mathematical errors at 3.76, strategy comparison and selection at 3.66, and pattern recognition and generalization at 3.60. These findings indicate that learners were regularly encouraged to explain solutions, consider different approaches, analyze mistakes, recognize mathematical relationships, and apply mathematical thinking to practical situations. The results show that teachers were especially inclined to promote explanation and real-life application, while strategy flexibility and generalization were comparatively less developed. These latter forms of reasoning require greater learner independence because pupils must evaluate alternative approaches and move from specific examples toward broader mathematical relationships. The overall result corroborates the synthesis of the proposal's literature and studies, which concludes that learner reasoning is strengthened when teachers guide pupils to explain, justify, compare strategies, recognize patterns, generalize ideas, analyze errors, and apply mathematics to real-life situations. The theoretical discussion likewise states that meaningful tasks, representations, questioning, scaffolding, collaboration, and formative assessment provide the classroom mechanisms through which reasoning develops.

#### **3.2.1 Explanation and Justification of Solutions**

Explanation and justification of solutions obtained an overall mean of 3.81, interpreted as Often. Asking learners to determine whether their answers made sense ranked highest at 4.01, followed by explaining how an answer was obtained at 3.96 and explaining through words, drawings, or symbols at 3.95. Lower ratings were recorded for improving incomplete explanations at 3.64 and presenting evidence supporting a solution at 3.58. The findings indicate that learners were regularly encouraged to explain their solution processes and assess the reasonableness of their answers. However, the progression from explanation toward more formal justification was less consistent, suggesting that learners had fewer opportunities to demonstrate why a method or answer was mathematically valid. The finding is consistent with the related literature emphasizing that explanation makes thinking visible while justification requires learners to provide reasons for the validity of a method or answer. Colonnese et al. (2026) likewise found that teacher prompting and opportunities for written justification strengthen the visibility and quality of learner reasoning.

#### **3.2.2 Strategy Comparison and Selection**

Strategy comparison and selection obtained an overall weighted mean of 3.66, interpreted as Often. Helping learners recognize that problems can be solved in more than one way ranked highest at 3.92, followed by allowing alternative solution methods at 3.81. Lower means were obtained for asking learners to explain why a particular strategy was selected at 3.58, encouraging them to change strategies after an unsuccessful attempt at 3.55, and comparing mental, written, visual, and concrete strategies at 3.41. The findings indicate that teachers generally accepted multiple methods but were less consistent in systematically teaching learners to compare, evaluate, and deliberately select among strategies. Strategic reasoning requires learners not only to know alternative procedures but also to determine their efficiency and suitability for particular problems. The findings correspond with the reviewed literature, which states that comparing solution strategies allows learners to identify similarities, differences, efficiency, and underlying mathematical structures. The broader literature in the proposal also stresses that learners require repeated opportunities for discussion, problem solving, representation, and reflection if strategic reasoning is to become part of ordinary mathematics learning.

#### **3.2.3 Pattern Recognition and Generalization**

Pattern recognition and generalization obtained an overall mean of 3.60, interpreted as Often, but was the lowest-rated learner-reasoning dimension. Predicting what comes next in a pattern received the highest mean of 3.82, followed by extending patterns at 3.75 and identifying patterns in numbers, shapes, or operations at 3.73. In contrast, comparing patterns across different examples obtained 3.46, while using patterns to solve unfamiliar mathematics problems and forming general statements from observed patterns obtained 3.39 and 3.35, respectively, both interpreted as Sometimes. The findings indicate that teachers regularly facilitated basic recognition and continuation of patterns but less consistently supported higher-order generalization. Moving from examples to general mathematical rules requires learners to identify invariant relationships, formulate and test rules, and apply them beyond familiar examples, making this an important area for strengthening early algebraic reasoning. The finding is supported by Johnson and Roy (2026), whose synthesis emphasized routines that promote generalization alongside multiple representations and conceptual reasoning. The proposal's related literature also notes that conjecturing and generalization require learners to notice, propose, test, and refine mathematical ideas, illustrating why these practices demand more sustained reasoning than simply completing a pattern.

#### **3.2.4 Reasoning Through Mathematical Errors**

Reasoning through mathematical errors obtained an overall mean of 3.76, interpreted as Often. Helping learners understand that mistakes can support deeper learning ranked highest at 4.08, followed by allowing learners to revise work after recognizing errors at 3.93 and using errors to clarify mathematical concepts at 3.89. Lower

means were obtained for comparing incorrect and correct solutions at 3.64, asking learners to explain how they improved answers after correction at 3.58, and using sample incorrect solutions to stimulate discussion at 3.48. The results indicate that teachers generally viewed errors constructively and allowed learners to improve their work, although the analytical examination of incorrect reasoning was less frequent. Greater use of incorrect solutions as objects of mathematical investigation could help learners identify misconceptions, explain where reasoning failed, and determine how solutions can be repaired. This interpretation is corroborated by the literature describing errors as windows into learner thinking and opportunities for reasoning rather than evidence of failure. Maboya et al. (2022) similarly found that mathematics teaching improved when teachers moved away from blaming learners for errors and instead reconsidered instructional practices, strengthened their content knowledge, and used manipulatives and reflection more effectively.

### **3.2.5 Transfer of Reasoning to Real-Life Problem Situations**

Transfer of reasoning to real-life problem situations obtained the highest overall learner-reasoning mean of 3.86. Problems involving time, money, measurement, or data received the highest mean of 4.15, followed by explaining the usefulness of mathematics in daily life at 4.10. Applying mathematical reasoning to everyday problems obtained 4.02, while checking the reasonableness of answers in real-life contexts received 3.95. However, asking learners to create real-life problems requiring mathematical reasoning obtained only 3.32, interpreted as Sometimes. The findings demonstrate that practical application was a prominent feature of learner reasoning facilitation and corresponded with the strong emphasis on contextualized mathematics instruction. Nevertheless, learners remained more frequently recipients than creators of contextualized problems. The result is strongly supported by the proposal's literature stating that real-life mathematical reasoning requires learners to formulate, employ, and interpret mathematics in practical situations. The reviewed literature on transfer likewise explains that transfer is not simple repetition of previous learning but recognition and use of mathematical structures across changing situations.

### **3.3 Relationship Between Mathematics Instructional Practices and Learner Reasoning Facilitation**

The analysis revealed a strong, positive, and statistically significant relationship between mathematics instructional practices and learner reasoning facilitation, with  $r = 0.74$  and  $p < .001$ . Thus, the null hypothesis stating that no significant relationship existed between the variables was rejected. Teachers reporting more frequent use of concept development, problem-based tasks, mathematical discourse, formative feedback, error analysis, and contextualization also tended to report higher levels of learner reasoning facilitation. The relationship was substantial but not perfect, indicating that other learner- and classroom-related factors may also contribute to reasoning facilitation. The result demonstrates that learner reasoning is closely associated with how mathematical content is presented, explored, discussed, assessed, and applied rather than functioning as a separate instructional activity. This pattern was also evident descriptively: contextualization and real-life transfer were the highest-rated dimensions within their respective variables, while formative feedback and error analysis corresponded with relatively strong reasoning through mathematical errors. The finding is corroborated directly by the related literature in the proposal, which argues that instruction involving concept development, representations, problem-based tasks, discourse, feedback, error analysis, and contextualization gives learners richer opportunities to explain, justify, compare, generalize, and transfer mathematical ideas. Auslander et al. (2023) likewise emphasized that mathematics teaching organized around learners' ideas and responsive instructional practice creates stronger conditions for reasoning facilitation.

### **3.4 Instructional-Practice Predictors of Learner Reasoning Facilitation**

The hierarchical multiple regression analysis showed that the five instructional-practice dimensions collectively explained 61 percent of the variance in learner reasoning facilitation, with an adjusted  $R^2$  of .57. Concept development and representation initially accounted for 39 percent of the variance and remained a significant predictor in the final model ( $\beta = .18$ ,  $p = .047$ ). Problem-based task structuring added 11 percent and was also significant ( $\beta = .25$ ,  $p = .011$ ), while mathematical discourse facilitation added another 8 percent and emerged as the strongest unique predictor ( $\beta = .32$ ,  $p = .003$ ). Formative feedback and error analysis increased cumulative  $R^2$  from .58 to .60 but was not a significant unique predictor ( $\beta = .13$ ,  $p = .144$ ), while contextualization increased  $R^2$  to .61 but likewise did not make a significant unique contribution ( $\beta = .09$ ,  $p = .276$ ). Consequently, the null hypothesis regarding the predictive effects of the instructional-practice dimensions was partially rejected. The results indicate that learner reasoning was particularly associated with practices that establish conceptual meaning, engage pupils in cognitively demanding tasks, and require them to express and examine mathematical thinking through discourse. The nonsignificant unique effects of formative feedback and contextualization do not indicate that these practices are unimportant; rather, their predictive contributions may overlap with the significant dimensions when considered simultaneously. Context becomes reasoning-rich when learners analyze, represent, discuss, and explain mathematical situations, while feedback may exert its strongest effects when integrated with challenging problems and discourse. The finding is corroborated by several studies in the proposal. Colonnese et al. (2026) showed that structured teacher prompts and discussion practices make learner reasoning visible. Ozturk (2025) likewise found that teacher talk moves involving eliciting ideas, introducing mathematical concepts, and encouraging learners to take positions were central to developing a reasoning-oriented classroom community.

Johnson and Roy (2026) further emphasized the combined importance of representation, conceptual coherence, discussion, and error analysis, supporting the conclusion that instructional practices work as an interconnected system rather than as isolated techniques.

### **3.5 Reasoning-Centered Mathematics Instruction Model**

The findings provided the basis for a Reasoning-Centered Mathematics Instruction Model comprising five interconnected components: Conceptual Anchoring and Representation, Cognitive Challenge Through Problem-Based Tasks, Reasoning Dialogue and Mathematical Discourse, Error-Feedback Reasoning Loop, and Contextual Transfer and Generalization. The model follows the recurring instructional process Represent → Challenge → Discuss → Analyze → Transfer. Conceptual anchoring establishes mathematical meaning through prior knowledge and multiple representations; problem-based tasks create intellectual challenge; discourse makes reasoning visible; error-feedback processes refine thinking; and contextual transfer and generalization extend learning beyond familiar classroom situations. The cycle is recursive rather than rigidly linear, allowing learners to revisit representations, reconsider strategies, analyze errors, and transfer revised reasoning to new situations. The model integrates the strongest predictors with the areas identified for further development. Mathematical discourse occupies a central role because it was the strongest unique predictor, while problem-based tasks and conceptual representation provide the intellectual foundation for reasoning. Feedback, error analysis, contextualization, strategy flexibility, transfer, and generalization serve as supporting processes that deepen mathematical thinking. The model is corroborated by the synthesis of the related literature, which emphasizes that mathematics learning should combine representation, problem solving, discourse, formative assessment, explanation, generalization, error analysis, and real-world application. It is further supported by Johnson and Roy (2026), who emphasized conceptual coherence, representations, error analysis, and discussion; Colonnese et al. (2026), who demonstrated the value of structured explanation and justification; and Schmidt et al. (2024), who showed that discourse facilitation can be intentionally developed through sustained pedagogical preparation.

## **4. Conclusion & Recommendations**

The study concludes that Key Stage 2 teachers in Ambaguio, Nueva Vizcaya generally practice reasoning-supportive mathematics instruction, particularly through contextualization, concept development and representation, and formative feedback, although problem-based task structuring requires greater emphasis on learner autonomy, exploration, and multiple solution approaches. Learner reasoning is regularly facilitated through real-life application, explanation and justification, and constructive use of errors, while strategy comparison, pattern recognition, and generalization are comparatively less developed. Mathematics instructional practices are strongly and positively related to learner reasoning facilitation, with mathematical discourse facilitation, problem-based task structuring, and concept development and representation emerging as significant predictors. These findings support the integration of conceptual understanding, challenging problem solving, mathematical discourse, error analysis, and meaningful application through the proposed Reasoning-Centered Mathematics Instruction Model using the recurring cycle Represent → Challenge → Discuss → Analyze → Transfer.

Key Stage 2 mathematics teachers should sustain their strengths in contextualization, concept development, representation, and formative feedback while strengthening problem-based tasks, strategy comparison, pattern recognition, generalization, and learner-generated mathematical problems. Greater emphasis should be placed on mathematical discourse, cognitively demanding problems, multiple representations, adequate exploration time, probing questions, peer interaction, and opportunities for learners to explain, defend, compare, and refine mathematical ideas. School heads and mathematics coordinators should support these practices through instructional supervision, coaching, Learning Action Cell sessions, collaborative lesson planning, demonstration teaching, and peer observation. The proposed Reasoning-Centered Mathematics Instruction Model may be flexibly used in lesson planning, classroom instruction, coaching, and professional development, while future research may examine and validate the model using intervention, experimental, quasi-experimental, mixed-method, or classroom-based studies in other grade levels and educational contexts.

The study was limited to Key Stage 2 teachers handling mathematics in Ambaguio, Nueva Vizcaya and focused specifically on their mathematics instructional practices and learner reasoning facilitation. It did not include respondent profiles, learner achievement scores, classroom observation ratings, or the perspectives of learners and parents. Consequently, the findings were based on the variables and respondents covered by the study and did not directly examine actual learner achievement or other possible influences on learner reasoning. Future investigations may address these limitations by incorporating additional evidence such as mathematics achievement, problem-solving performance, mathematical communication, learner engagement, classroom observations, learner-generated solutions, and interviews, as well as by examining the model in other districts, municipalities, grade levels, and educational contexts.

## 5. Compliance with ethical standards

The study adhered to ethical standards intended to protect the rights, dignity, privacy, and welfare of the respondents. Appropriate permission was secured from the concerned division and school authorities before data collection, and participation was voluntary, with respondents informed of the study's purpose and their right to decline or withdraw without penalty. Informed consent was obtained before questionnaire administration, while confidentiality and anonymity were maintained throughout the analysis, interpretation, and reporting of findings. All collected information was used solely for academic and research purposes, handled securely and objectively, and managed in a manner that avoided unnecessary physical, psychological, professional, or social risk. The collection, storage, analysis, and reporting of data were conducted with integrity and without fabrication, falsification, alteration, or manipulation of information.

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