

Conceptual Mathematics Teaching and Learner Reasoning Support of Key Stage 2 Teachers in Ambaguio, Nueva Vizcaya

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Abstract

This study examined conceptual mathematics teaching practices, learner reasoning support, and classroom mathematical discourse facilitation among Key Stage 2 teachers in Ambaguio, Nueva Vizcaya, and developed an instructional enhancement framework based on the findings. Anchored in constructivist learning theory, social constructivist learning theory, mathematical proficiency theory, pedagogical content knowledge, and teacher noticing, the study employed a quantitative explanatory pathway design with predictive mediation modeling. Sixty-four Grades 4–6 teachers were selected through simple random sampling and surveyed using a researcher-made questionnaire with an overall Content Validity Index of 0.94 and Cronbach's alpha of 0.95. Conceptual mathematics teaching, learner reasoning support, and classroom mathematical discourse facilitation were generally practiced to a great extent. However, learner-generated representations, structured error analysis, strategy comparison, peer mathematical discussion, misconception reconstruction, and reasoning-extending feedback required strengthening. Conceptual mathematics teaching was strongly and positively related to learner reasoning support ($r = .671$, $p < .001$). The final regression model explained 55.2% of the variance in reasoning support, with conceptual error analysis ($\beta = .35$, $p = .001$) and concept connection and progression ($\beta = .27$, $p = .010$) emerging as significant predictors. Classroom mathematical discourse partially mediated the relationship, with a significant indirect effect of .33, 95% CI [.18, .49]. The study recommends strengthening learner-centered reasoning and discourse practices through the proposed instructional enhancement framework and school-based professional learning. The study aligns with SDG 4—Quality Education by supporting responsive, conceptually grounded mathematics instruction and teacher professional development. Its sustainability impact is primarily educational, socio-cultural, and institutional, supporting durable mathematical understanding and locally informed improvement of teaching practice.

Keywords: Classroom Mathematical Discourse Facilitation, Conceptual Error Analysis, Conceptual Mathematics Teaching, Instructional Enhancement Framework, Key Stage 2, Learner Reasoning Support, Mathematical Language, Mathematics Education, Teacher Professional Development

1. Introduction

Mathematics education in Key Stage 2 increasingly requires learners to move beyond procedural recall toward conceptual understanding, reasoning, communication, problem solving, and application. The present study examines this instructional shift through three connected constructs among Key Stage 2 teachers in Ambaguio, Nueva Vizcaya: conceptual mathematics teaching practices as the predictor variable, learner reasoning support as

the outcome variable, and classroom mathematical discourse facilitation as a possible mediating mechanism. The study is situated within Philippine curriculum expectations and broader evidence of persistent difficulties in mathematical proficiency, while drawing on literature that emphasizes conceptual connections, multiple representations, contextualization, error analysis, purposeful questioning, justification, strategy comparison, feedback, and mathematical discourse.

Contemporary elementary mathematics teaching emphasizes understanding why mathematical ideas work, how concepts connect, and how mathematics can be used to explain situations and justify solutions. The Department of Education [DepEd] (2023a) emphasized that Key Stage 2 learners should use mathematical concepts, operations, procedures, relationships, and tools in routine and non-routine real-world problems and communicate reasoning using precise mathematical language. Consistent with this direction, DepEd (2023b) described the MATATAG Mathematics curriculum as aiming to develop mathematically proficient and critical problem solvers grounded in conceptual knowledge, strategic use of mathematical skills and processes, desirable values, and appropriate mathematical disposition. Conceptual mathematics teaching helps learners connect prior and new ideas, use varied representations, contextualize concepts, and analyze errors rather than merely memorize procedures. Learner reasoning support complements this by prompting learners to explain, justify, compare strategies, identify errors, and extend mathematical thinking. The OECD (2023a) positioned mathematical reasoning as central to mathematical literacy and problem solving, reinforcing the need for classrooms in which learners explain, test, revise, and communicate ideas rather than focus only on answer production. Teacher expertise is therefore central: Copur-Gençtürk (2022) identified content knowledge, pedagogical content knowledge, and content-specific noticing as distinct dimensions of elementary mathematics teaching expertise; Aderinoye-Rabiu et al. (2025) emphasized the contribution of content and pedagogical content knowledge to teaching effectiveness; and Moremi (2025) showed that noticing learner thinking may remain superficial when instructional responses do not engage deeply with learners' reasoning. The instructional challenge is particularly relevant in Ambaguio, Nueva Vizcaya, where mathematics teaching may need to respond to varied geographic, linguistic, and socio-cultural experiences. Contextualized examples can make mathematical ideas accessible, but effective instruction must still move learners from concrete experience to representation, mathematical language, abstraction, and justification. Vistro-Yu, Hao, and Sua (2025) observed that the Philippine mathematics curriculum contains opportunities for problem posing even when it is not explicitly articulated as a teaching strategy or competency, suggesting the need for teacher support in practices that extend beyond routine exercises. Lit (2026) likewise emphasized teacher knowledge and noticing as central to mathematical problem-solving instruction.

The literature consistently treats conceptual mathematics teaching as instruction organized around meaning, relationships, structure, and transfer rather than isolated procedures (Ncube & Luneta, 2025). Concept connection and progression require coherent links across prerequisite, current, and future ideas (Department of Education, 2023a), while multiple representations support understanding when learners translate meaning across symbolic, visual, concrete, verbal, and contextual forms (Zentgraf & Prediger, 2024). Contextualization is strongest when familiar situations become bridges toward mathematization and abstraction (Wei et al., 2022), and conceptual error analysis uses mistakes as evidence of learner thinking rather than as occasions for simple correction (Shimizu & Kang, 2025). Conceptual and procedural knowledge are therefore complementary rather than competing forms of mathematical competence (Ncube & Luneta, 2025). Conceptual learning is strengthened by tasks that reveal mathematical structure and require purposeful activity and reflection (Moreno et al., 2024). Representations become explanatory when learners understand what they mean and how they connect to notation (Zentgraf & Prediger, 2024), while contextualized tasks become mathematically productive when experience is progressively organized into formal relationships (Wei et al., 2022). Error analysis also functions as formative assessment when learner responses guide instructional adjustment (Söderström & Palm, 2024). Cross-domain connections help learners see mathematics as an integrated discipline (Department of Education, 2023b), and varied representations can broaden conceptual access for learners with different strengths (CAST, 2024). Effective contextualization must reveal mathematical structure rather than merely add local details (Abdelrahim et al., 2025), while curriculum coherence supports increasingly sophisticated learning across lessons and grade levels (Department of Education, 2023a). From a cognitive and explanatory perspective, conceptual instruction supports durable schemas and transfer (Ncube & Luneta, 2025), requires diagnostic distinctions among procedural, conceptual, linguistic, and representational errors (Shimizu & Kang, 2025), and uses explanation as a generative learning process (Ncube & Luneta, 2025). Comparing representations supports abstraction by making invariant mathematical structure visible across forms (Zentgraf & Prediger, 2024), while mathematical modeling connects meaningful situations to mathematical structures and evaluation (Cevikbas et al., 2022). Ultimately, conceptual understanding becomes visible when learners can explain, represent, connect, apply, and revise their thinking (Ncube & Luneta, 2025). Learner reasoning support shifts attention from producing answers to making mathematical thinking explicit, logical, and defensible (Stylianides et al., 2024). Reasoning-oriented questioning makes thought processes visible and positions learners as active meaning-makers (Öz & Işık, 2024). Explanation and justification establish why mathematical claims are acceptable (Sjöblom et al., 2023), comparison of solution strategies develops flexibility and metacognition (Öz & Işık, 2024), and feedback extends reasoning when it

prompts clarification, revision, representation, or generalization (Söderström & Palm, 2024). These practices depend on classroom norms that value thinking, respectful challenge, and justification over speed (Sjöblom et al., 2023). Reasoning support must also be responsive. Effective questions build from learners' emerging ideas (Lee, 2025), while mathematical language helps learners move from informal expression toward precise conceptual communication (Planas & Pimm, 2023). Age-appropriate justification can develop through models, drawings, examples, and logical explanations (Stylianides et al., 2024), and strategy comparison can reveal the structures behind procedures (Öz & Işık, 2024). Feedback is most useful when it identifies the next mathematical thinking move rather than merely evaluates correctness (Söderström & Palm, 2024). Problem solving provides a natural setting for interpretation, planning, strategy selection, and evaluation (Rahman, 2023), while probing, pressing, redirecting, and extending questions distribute reasoning across classroom interaction (Sjöblom et al., 2023). Learners also need opportunities to distinguish examples from general reasoning (Öz & Işık, 2024), exercise agency in choosing and defending strategies (Rahman, 2023), and translate reasoning across representations (Mayer, 2021). Anticipating learner responses helps teachers prepare productive follow-up questions and discussion sequences (Sjöblom et al., 2023), while sufficient wait time supports deeper processing and communicates that thoughtful responses matter (Lee, 2025). Reasoning is further strengthened when classrooms normalize uncertainty, revision, and productive struggle (Rahman, 2023), consistent with the emphasis on reasoning in the PISA mathematics framework (Organisation for Economic Co-operation and Development, 2023a). Classroom mathematical discourse facilitation provides the social mechanism through which conceptual understanding and reasoning become public, examinable, and revisable. Productive discourse requires teachers to orchestrate participation and sustain mathematically meaningful talk (Sjöblom et al., 2023), scaffold learner contributions without taking over the reasoning process (Sjöblom et al., 2023), and support exploratory talk in which ideas are jointly examined (Planas & Pimm, 2023). Mathematical language is itself a resource for meaning-making (Planas & Pimm, 2023), and language-responsive instruction can preserve cognitive demand while supporting access (Sharma & Sharma, 2023). Dialogic teaching further treats classroom talk as a cumulative process for developing ideas (Planas & Pimm, 2023). Teacher noticing is critical to discourse because teachers must attend to, interpret, and respond to learner thinking (Weyers et al., 2024). Productive classroom communities also depend on norms that authorize learners to contribute and critique mathematical ideas (Sjöblom et al., 2023), particularly in multilingual settings where language resources influence participation (Sharma & Sharma, 2023). Misconceptions can become resources for conceptual change when they are surfaced and discussed rather than simply corrected (Shimizu & Kang, 2025). Purposeful sequencing of learner responses can create coherent mathematical discussions (Sjöblom et al., 2023), while cognitively demanding tasks require facilitation that preserves opportunities for reasoning (Moreno et al., 2024). Talk moves and discussion routines can help teachers elicit and connect learner ideas (Sjöblom et al., 2023). Across these themes, representations, discourse, and reasoning are mutually reinforcing. Visual and symbolic forms can support mathematical argument when learners explain what they show (Zentgraf & Prediger, 2024); rich tasks require discussion structures that sustain their cognitive demand (Moreno et al., 2024); and equitable discourse depends on how authority and participation are distributed (Sjöblom et al., 2023). Culturally responsive mathematics instruction connects learner identities and experiences to rigorous mathematical activity (Abdelrahim et al., 2025), while classroom argumentation develops when learners evaluate claims and evidence (Sjöblom et al., 2023). Mathematical language, gesture, and representation work together in communicating meaning (Planas & Pimm, 2023), and collaborative interaction can make reasoning visible and revisable (Sjöblom et al., 2023). The relationship between conceptual teaching and reasoning is also supported by work on understanding, transfer, and teacher practice. Meaningful mathematical learning depends on making conceptual relationships explicit (Ncube & Luneta, 2025), while effective teaching requires attention to both mathematical meaning and learner sense-making (Ncube & Luneta, 2025). Conceptual teaching and reasoning are strengthened when instruction emphasizes connected understanding rather than isolated procedures (Mainali, 2021), supports mathematization from realistic situations (Wei et al., 2022), and provides opportunities for learners to explain why methods work (Shimizu & Kang, 2025). Teacher professional competence influences the quality of such instructional decisions (Aderinoye-Rabiu et al., 2025), and transfer is more likely when learners recognize structural similarities across problems (Ncube & Luneta, 2025). Teachers therefore need specialized knowledge to select representations, interpret responses, and support reasoning. Elementary teaching expertise includes content-specific noticing (Copur-Gencurk, 2022), while productive discussions depend on selecting and sequencing tasks and responses that advance mathematical goals (Sjöblom et al., 2023). Teachers must recognize the affordances and limitations of representations (Zentgraf & Prediger, 2024), connect contextual problems with formal mathematical modeling (Cevikbas et al., 2022), and use counterexamples and conceptual conflict to refine understanding (Shimizu & Kang, 2025). Responsive teaching also depends on interpreting the substance of learner strategies (Jacobs et al., 2024). Conceptual teaching makes underlying structure visible (Ncube & Luneta, 2025), productive struggle can deepen sense-making when appropriately supported (Rahman, 2023), and feedback can connect understanding with explanation and justification (Söderström & Palm, 2024). Metacognitive reflection strengthens awareness of strategies and conceptual gaps (Wei et al., 2022), generalization moves learners from cases to broader relationships (Stylianides et al., 2024), and mathematical discourse carries conceptual understanding into visible reasoning practice (Sjöblom et al., 2023). Together, these practices support meaningful mathematical competence (Organisation for Economic Co-operation and Development, 2023a).

At the national level, the Organisation for Economic Co-operation and Development [OECD] (2023b) reported that only 16 percent of Filipino students reached at least Level 2 proficiency in mathematics in PISA 2022, compared with an OECD average of 69 percent, while almost no Filipino students reached the highest proficiency levels. This evidence points to difficulties not only in computation but also in interpretation, modeling, reasoning, and strategic problem solving. At the curriculum level, Philippine expectations explicitly emphasize conceptual knowledge, problem solving, reasoning, communication, and application, making classroom enactment a central concern. At the local level, the study focuses on Key Stage 2 teachers in Ambaguio, Nueva Vizcaya. The setting makes contextualization, mathematical language, learner participation, and sensitivity to varied experiences especially relevant to instruction. The literature reviewed from international settings similarly shows that conceptual teaching and reasoning are shaped by teacher preparation, professional learning, language, inclusion, technology, task design, and classroom culture. Zhang et al. (2025) showed how purposeful questioning can extend conceptual and spatial reasoning in play-based contexts; Priyanda et al. (2025) emphasized that technology supports conceptual learning only when integrated with pedagogical and content knowledge; and Padilla et al. (2025) highlighted pedagogical content knowledge as central to meaningful early mathematics instruction. Across other contexts, Ncube and Luneta (2025) reported benefits of concept-based instruction for understanding, critical thinking, retention, transfer, and engagement, while Davidson et al. (2025) showed the potential of teacher-led professional learning for sustaining reasoning-focused pedagogy. Jiménez-Silva et al. (2025) emphasized the integration of language, mathematical thinking, and cultural identity for multilingual learners, and Abdelrahim et al. (2025) showed how culturally meaningful literature can provide contexts for mathematical problem solving and discourse. Inclusive access is also important: Maroba and Mphunyane (2025) identified the need for specialized pedagogical and communication support for hearing-impaired learners, while Mutia (2025) linked persistent mathematical difficulties to weak foundations, anxiety, ineffective instruction, and limited conceptual understanding. Teacher preparation studies reinforce the gap between valuing conceptual and reasoning-centered instruction and being able to enact it. Suryanti and Yanti (2025) found that pre-service teachers valued conceptual fraction teaching and visual models but often lacked implementation confidence. Moreno et al. (2024) showed that inquiry-based teaching becomes stronger when task design is explicitly connected to intended reasoning practices, while Aziz et al. (2024) found weaknesses in prospective teachers' conceptual validity, proof, mathematical connections, and communication. Teng and Yang (2024) identified barriers involving conceptual systems, mathematical language, technology integration, learner prior knowledge, and professional guidance. Other studies show practical routes for strengthening conceptual and reasoning-centered instruction. Bohara (2024) found that project-based learning can support contextualized and conceptually meaningful mathematics pedagogy; Gunawan and Hadi (2024) reported that Realistic Mathematics Education improved conceptual and procedural understanding when connected with reasoning; and Öz and Işık (2024) found that opportunities for mathematical reasoning may remain too limited unless teachers deliberately build explanation, justification, comparison, and generalization into lessons. Bogota (2024) showed that translanguaging can support reasoning and sense-making among language-minoritized students. Lo (2024) found that K–6 teacher candidates require structured professional development to move from skill repetition toward reform-based pedagogy, while Kanandjebo (2024) highlighted the need for responsive pedagogy and accessible representations for learners with visual and hearing impairments. Weingarden and Buchbinder (2023) further demonstrated that lesson planning and revision can strengthen prospective teachers' capacity to teach through reasoning and proving.

The study is anchored in constructivist learning theory, social constructivist learning theory, mathematical proficiency theory, pedagogical content knowledge, and teacher noticing. Constructivism views learners as active constructors of knowledge who connect new information with prior understanding, test ideas, identify relationships, and reorganize thinking. Munir (2025) described knowledge as actively constructed rather than passively transmitted, supporting instruction that allows learners to explore patterns, manipulate objects, compare examples, use representations, and explain procedures. Social constructivism emphasizes interaction, language, and guided participation. Mangwende (2026) explained that knowledge is actively constructed through interaction with others and the environment. This perspective directly supports learner reasoning through classroom discussion, explanation, comparison, questioning, and teacher prompts. Mathematical proficiency theory complements these perspectives by emphasizing balanced development of conceptual knowledge, strategic skills, reasoning, communication, and productive disposition; DepEd (2023b) reflects this orientation in its goal of mathematically proficient and critical problem solvers. Pedagogical content knowledge explains the professional capacity to transform mathematical subject matter into forms learners can understand. Aderinoye-Rabiu, Bello, Usman, Falebita, and Oloyede (2025) found that pedagogical content knowledge is significantly associated with teaching effectiveness among primary mathematics educators. Teacher noticing further explains how teachers attend to learner thinking, interpret its mathematical meaning, and decide how to respond; Copur-Gencturk (2022) and Moremi (2025) underscore the importance of such noticing for instructional decisions. Conceptually, the study treats conceptual mathematics teaching practices as the predictor variable, learner reasoning support as the outcome variable, and classroom mathematical discourse facilitation as the mediating variable. Conceptual mathematics teaching comprises concept connection and progression, multiple mathematical representations, contextualization of mathematical ideas, and facilitation of conceptual error analysis. Learner reasoning support

comprises reasoning-oriented questioning, learner explanation and justification, comparison of solution strategies, and feedback that extends mathematical thinking. Classroom mathematical discourse facilitation comprises teacher prompting and probing, learner-to-learner mathematical discussion, use of mathematical language, and productive handling of misconceptions. The proposed pathway is that conceptual teaching may strengthen reasoning directly and may also do so through richer mathematical discourse. This framing is consistent with the OECD (2023a) emphasis on reasoning as central to mathematical literacy and problem solving.

The reviewed literature establishes strong theoretical and empirical support for conceptual mathematics teaching, learner reasoning support, and classroom mathematical discourse, but it also reveals an implementation gap. Teachers may value conceptual understanding yet struggle to connect concepts, select and coordinate representations, diagnose misconceptions, use precise mathematical language, sustain cognitively demanding discussion, or respond deeply to learner thinking. Studies also show that reasoning opportunities can remain limited even when teachers intend to support them, and that professional preparation, language, inclusion, contextualization, and instructional resources affect enactment. A more specific gap concerns the relationship among the three constructs in the local Key Stage 2 context. Existing work supports conceptual teaching, reasoning, and discourse individually and suggests that discourse can carry conceptual understanding into visible reasoning, but the uploaded review does not present evidence establishing how these variables operate together among Key Stage 2 teachers in Ambaguio, Nueva Vizcaya. The study therefore examines the extent of conceptual mathematics teaching practices and learner reasoning support, the level of classroom mathematical discourse facilitation, the relationship and predictive contribution of conceptual teaching to reasoning support, and whether discourse facilitation mediates that relationship. The resulting evidence is intended to ground an instructional enhancement framework responsive to observed classroom strengths and needs.

The study most directly aligns with SDG 4 – Quality Education because its central concern is the quality of mathematics teaching and the development of meaningful learning in Key Stage 2. Its focus on conceptual understanding, reasoning, mathematical communication, inclusive access to representations and discourse, and teacher professional development is consistent with improving the quality and responsiveness of educational practice. The study's proposed instructional enhancement framework may support this alignment by using local evidence to strengthen teaching practices rather than relying only on generalized prescriptions.

The study contributes primarily to educational sustainability by examining teaching practices that can support durable mathematical understanding, reasoning, and transfer across grade levels. It also has socio-cultural relevance because contextualization, multilingual participation, and attention to learners' varied experiences can make mathematical learning more accessible without reducing conceptual rigor. At the institutional level, findings may inform school-based coaching, Learning Action Cell activities, instructional supervision, curriculum implementation, and professional development for Key Stage 2 mathematics teachers. These contributions are grounded in the study's existing focus on teacher practice, learner reasoning, classroom discourse, and an instructional enhancement framework; the study does not directly investigate environmental or ecological sustainability.

The study is delimited to Key Stage 2 teachers in Ambaguio, Nueva Vizcaya. It examines conceptual mathematics teaching practices through concept connection and progression, use of multiple mathematical representations, contextualization of mathematical ideas, and facilitation of conceptual error analysis; learner reasoning support through reasoning-oriented questioning, learner explanation and justification, comparison of solution strategies, and feedback that extends mathematical thinking; and classroom mathematical discourse facilitation through teacher prompting and probing, learner-to-learner mathematical discussion, use of mathematical language, and productive handling of misconceptions. It further examines relationships, prediction, and mediation among these constructs and uses the findings as the basis for an instructional enhancement framework. The study does not directly measure learner achievement scores, standardized test performance, or long-term outcomes, and its findings are interpreted within the Key Stage 2 context of Ambaguio, Nueva Vizcaya.

2. Material and methods

2.1 Research Design

The study used a quantitative explanatory pathway design with predictive mediation modeling. This design described the extent of conceptual mathematics teaching practices, learner reasoning support, and classroom mathematical discourse facilitation while examining their relationships. Specifically, it determined whether conceptual mathematics teaching practices directly influenced learner reasoning support, identified which dimensions significantly predicted such support, and tested whether classroom mathematical discourse facilitation mediated the relationship between the two constructs. The resulting evidence served as the basis for developing an instructional enhancement framework for Key Stage 2 mathematics instruction.

2.2 Locale of the Study

The study was conducted in schools offering Key Stage 2 instruction in Ambaguio, Nueva Vizcaya. The locale provided an appropriate setting for examining how elementary mathematics teachers facilitated conceptual understanding, mathematical representations, discourse, and learner reasoning during the transition from basic numeracy to more complex mathematical thinking. The investigation focused specifically on teachers' instructional practices, discourse facilitation, and learner reasoning support rather than respondent profile variables.

2.3 Respondents of the Study

The respondents were 64 Key Stage 2 teachers who handled Mathematics or Mathematics-related competencies in Grades 4, 5, and 6 in Ambaguio, Nueva Vizcaya. They were included because of their direct involvement in teaching mathematical concepts, developing learners' reasoning, and facilitating mathematical discourse at the upper elementary level. Learners, school heads, parents, and other school personnel were excluded because the investigation specifically addressed teachers' instructional practices. No demographic or professional profile variables were collected because the study did not seek to compare respondents according to these characteristics.

2.4 Sample and Sampling Procedure

Simple random sampling was used to select the 64 respondents. A complete list of qualified Key Stage 2 teachers was obtained from the concerned schools and appropriate education offices, after which respondents were randomly selected. This procedure gave every eligible teacher an equal opportunity for inclusion, thereby minimizing selection bias and promoting fair representation of the target population.

2.5 Research Instrument

Data were collected using a researcher-made survey questionnaire titled Conceptual Mathematics Teaching and Learner Reasoning Support Questionnaire, developed according to the research questions and principal study variables. The instrument measured conceptual mathematics teaching practices through concept connection and progression, use of multiple mathematical representations, contextualization of mathematical ideas, and facilitation of conceptual error analysis. Learner reasoning support was assessed through reasoning-oriented questioning, learner explanation and justification, comparison of solution strategies, and feedback that extends mathematical thinking. Classroom mathematical discourse facilitation was measured through teacher prompting and probing, learner-to-learner mathematical discussion, use of mathematical language, and productive handling of misconceptions. A five-point Likert scale measured the extent to which these practices were observed. Mean scores from 4.21 to 5.00 indicated a Very Great Extent, 3.41 to 4.20 a Great Extent, 2.61 to 3.40 a Moderate Extent, 1.81 to 2.60 a Low Extent, and 1.00 to 1.80 a Very Low Extent. Content validation was conducted by five experts in mathematics education, educational research, educational measurement, and elementary education. They assessed the items for content, clarity, relevance, organization, and alignment with the Statement of the Problem and study constructs. Their recommendations were incorporated into the revised questionnaire, resulting in an overall Content Validity Index of 0.94. The instrument was subsequently pilot tested with 15 Key Stage 2 teachers from a nearby locality who were excluded from the actual respondents, leading to minor revisions based on comprehensibility, appropriateness, and internal consistency. Reliability was assessed using Cronbach's Alpha. The Conceptual Mathematics Teaching Practices scale obtained a coefficient of 0.93, the Learner Reasoning Support scale 0.94, and the Classroom Mathematical Discourse Facilitation scale 0.92. The overall questionnaire obtained a coefficient of 0.95, indicating excellent internal consistency and supporting its use in the actual data collection.

2.6 Data Gathering Procedure

The researcher first obtained permission from the appropriate education offices and school authorities and coordinated the schedule and manner of data collection with the participating schools. Before final administration, the questionnaire underwent expert validation and pilot testing, followed by reliability analysis and necessary revisions. During data collection, respondents were informed of the study's purpose, the voluntary nature of their participation, and the confidentiality of their responses. They were provided sufficient time to complete the questionnaire independently and honestly. Completed questionnaires were retrieved and checked for completeness before responses were coded and prepared for analysis. Incomplete or inconsistent responses were screened using established data-cleaning procedures. The encoded data were organized according to the study variables and domains. Descriptive analyses were conducted first, followed by relationship, prediction, and mediation analyses. The resulting findings provided the basis for developing the proposed instructional enhancement framework.

2.7 Data Analysis Techniques

Weighted mean and standard deviation were used to determine the extent of conceptual mathematics teaching practices, learner reasoning support, and classroom mathematical discourse facilitation and to describe the consistency or variation in teachers' responses. Pearson Correlation Coefficient was employed to determine the significance, strength, and direction of the relationship between conceptual mathematics teaching practices and learner reasoning support. Hierarchical Multiple Regression Analysis identified which dimensions of conceptual mathematics teaching practices significantly predicted learner reasoning support and assessed their respective

predictive contributions. Bootstrapped Mediation Analysis determined whether classroom mathematical discourse facilitation mediated the relationship between conceptual mathematics teaching practices and learner reasoning support by estimating the indirect effect and its confidence intervals. Findings from the descriptive, correlational, regression, and mediation analyses were subsequently synthesized through results-based framework mapping to identify areas requiring strengthening, dimensions with stronger predictive value, and statistically supported instructional pathways. This synthesis guided the development of the instructional enhancement framework for conceptual mathematics teaching and learner reasoning support.

2.8 Ethical Considerations

Ethical standards were observed throughout the research process. Approval was obtained from the appropriate authorities, and respondents were informed of the study's purpose, the nature of their participation, and their right to refuse or withdraw without negative consequences. Informed consent was secured before participation, and no sensitive personal information or profile variables were collected. Confidentiality was maintained by withholding the identities of respondents and schools from the presentation of findings and using responses solely for academic and research purposes. Data were encoded, stored, and reported in summary form to prevent individual identification. The researcher further committed to accurate and honest reporting without altering, fabricating, or misrepresenting data, with interpretations guided by the research questions, statistical findings, and the ethical responsibility to represent mathematics teaching practices fairly and respectfully.

3. Results and Discussion

3.1 Conceptual Mathematics Teaching Practices

3.1.1 Concept Connection and Progression

Concept connection and progression was practiced to a Great Extent, with an overall weighted mean of 3.91 and an average standard deviation of 0.65. Sequencing mathematics activities from simple to complex concepts obtained the highest mean of 4.15, followed by revisiting prerequisite concepts at 4.10 and connecting new lessons with previously studied concepts at 4.08. The lowest-rated practices were using lesson transitions to demonstrate how one mathematical idea leads to another (3.68) and explaining how current topics contribute to future lessons (3.72), although both remained within Great Extent. These findings indicate that teachers generally recognize mathematics as cumulative and deliberately connect instruction with learners' prior knowledge. However, explicit communication of the conceptual pathway between present and future topics is comparatively less consistent. The result agrees with the related literature describing concept connection and progression as deliberate linking of prior, present, and future mathematics through prerequisite knowledge and conceptual bridges rather than isolated procedural learning.

3.1.2 Use of Multiple Mathematical Representations

The use of multiple mathematical representations obtained an overall mean of 3.78, interpreted as Great Extent. Teachers frequently used drawings, diagrams, or visual models (4.09), concrete materials (4.03), and representations for translating word problems into equations, drawings, or models (4.00). In contrast, asking learners to demonstrate solutions in more than one way received 3.38, interpreted as Moderate Extent, while allowing learners to choose representations that best communicated their thinking received 3.41. The pattern suggests that teachers effectively provide representations but give learners fewer opportunities to independently generate, compare, and select them. This distinction is important because conceptual understanding is strengthened when learners actively move among symbolic, visual, concrete, and verbal representations. The findings agree with Priyanda et al. (2025), who emphasized purposeful connections among digital, symbolic, verbal, and visual representations, and Suryanti and Yanti (2025), who recommended sustained teacher preparation in area models, number lines, visual explanations, and related representations.

3.1.3 Contextualization of Mathematical Ideas

Contextualization emerged as the strongest conceptual mathematics teaching domain, with an overall mean of 4.03. Relating mathematics lessons to learners' everyday experiences obtained the highest mean of 4.24, interpreted as Very Great Extent. Using familiar situations (4.12) and explaining the usefulness of mathematics outside the classroom (4.11) were also strongly practiced. The use of local or community-based examples obtained the lowest mean of 3.88, although still interpreted as Great Extent. The findings show that teachers commonly connect mathematics with practical and recognizable experiences, although locally specific contexts are somewhat less systematically incorporated. Contextualization is most meaningful when familiar experiences lead learners toward formal mathematical relationships and abstraction. This interpretation is consistent with Abdelrahim et al. (2025), who showed that culturally meaningful contexts can strengthen mathematical problem solving, discourse, and conceptual understanding when lived experiences are deliberately connected with mathematical relationships.

3.1.4 Facilitation of Conceptual Error Analysis

Conceptual error analysis was practiced to a Great Extent, with an overall mean of 3.79. Discussing common errors without embarrassing learners received the highest mean of 4.23, interpreted as Very Great Extent, while

encouraging learners to regard errors as part of learning obtained 4.02. In contrast, using structured error-analysis activities to strengthen conceptual understanding obtained 3.39, interpreted as Moderate Extent. Comparing incorrect solutions with correct reasoning (3.64) and helping learners identify where misunderstanding occurred (3.68) were also comparatively lower. Teachers therefore appear stronger in maintaining a supportive climate around mistakes than in systematically using errors as objects of mathematical investigation. More structured error analysis can make misconceptions visible and require learners to diagnose, explain, and revise their reasoning. This interpretation corresponds with Mutia (2025), whose study associated persistent mathematical difficulties with weak conceptual foundations and recommended diagnostic intervention, reasoning questions, explanation of errors, and revision of strategies.

3.2 Learner Reasoning Support

3.2.1 Reasoning-Oriented Questioning

Reasoning-oriented questioning obtained an overall mean of 3.78, indicating Great Extent. Encouraging learners to explain mathematical ideas in their own words received the highest mean of 4.10, followed by identifying patterns or relationships at 4.02 and explaining why answers are correct at 3.94. Asking what would happen if a number, condition, or rule changed received 3.38, interpreted as Moderate Extent, while requiring evidence for mathematical claims obtained 3.56. The findings indicate that teachers commonly elicit explanations but less consistently employ questions requiring learners to extend, test, or generalize mathematical relationships. Zhang et al. (2025) found that purposeful “why,” “how,” and “what changes if” questions expand opportunities for mathematical reasoning and negotiation of ideas. Similarly, Öz and Işık (2024) observed that reasoning opportunities depend on prompts requiring learners to justify, compare, and generalize rather than stopping after a correct answer.

3.2.2 Learner Explanation and Justification

Learner explanation and justification was the strongest reasoning-support dimension, with an overall mean of 3.85, interpreted as Great Extent. Providing opportunities for learners to explain how they solved problems received 4.06, followed by explaining solution steps at 3.99 and providing time to organize thoughts before explaining at 3.96. Encouraging revision of incomplete explanations received the lowest mean of 3.64, followed by clarifying incomplete mathematical statements at 3.69 and connecting explanations with mathematical concepts at 3.70. The results suggest that teachers frequently provide opportunities for explanation but are somewhat less consistent in pressing learners to refine explanations into complete mathematical justifications. Aziz et al. (2024) showed that mathematical responses may remain partially valid when claims lack sound reasons, connections, and coherent communication. Moreno et al. (2024) likewise emphasized tasks and interactions that require learners to investigate, explain, justify, and connect mathematical ideas.

3.2.3 Comparison of Solution Strategies

Comparison of solution strategies obtained an overall mean of 3.66, interpreted as Great Extent, but was the lowest learner reasoning-support domain. Allowing learners to share alternative solutions obtained the highest mean of 3.97, followed by presenting multiple ways of solving problems at 3.89. Using discussions to examine the strengths and limitations of strategies received 3.36, while assessing whether a strategy could transfer to other problems obtained 3.39; both were interpreted as Moderate Extent. The findings indicate that alternative strategies are more frequently presented or shared than critically compared for efficiency, generalizability, strengths, and limitations. Reasoning-oriented instruction therefore requires moving beyond multiple-solution sharing toward explicit evaluation of mathematical approaches. This supports the literature identifying strategy comparison as important to reasoning and corresponds with Öz and Işık (2024), who found that reasoning is strengthened when learners compare and generalize mathematical ideas.

3.2.4 Feedback That Extends Mathematical Thinking

Feedback that extends mathematical thinking was practiced to a Great Extent, with an overall mean of 3.73. Pointing out what learners did correctly before guiding improvement received the highest mean of 4.05. Feedback encouraging deeper explanation and improved reasoning obtained 3.88 and 3.84, respectively. Encouraging learners to attempt another solution strategy received 3.38, interpreted as Moderate Extent, while follow-up questioning after feedback obtained 3.42. These results show that feedback is predominantly constructive but less consistently used to initiate another cycle of reasoning. Effective reasoning-oriented feedback should extend dialogue through revision, alternative strategies, comparison, and further justification. Öz and Işık (2024) recommend continuing questioning after initial responses so that learners explain, compare, and generalize, while Davidson et al. (2025) found value in professional learning that shifts questioning from answer-focused exchanges toward explanation and analysis of learner thinking.

3.3 Classroom Mathematical Discourse Facilitation

3.3.1 Teacher Prompting and Probing

Teacher prompting and probing obtained an overall mean of 3.79, interpreted as Great Extent. Guiding prompts when learners struggled to explain their thinking received 3.93, probing questions for clarification received 3.91,

and questions about mathematical patterns and relationships obtained 3.90. Asking learners to reconsider incomplete reasoning had the lowest mean of 3.61, followed by prompting learners to use examples or models at 3.68. The findings suggest that questioning commonly functions as instructional scaffolding, although probing could be directed more consistently toward revision and evidentiary support. This agrees with the literature describing prompting and probing as discourse moves that encourage elaboration, clarification, justification, and reconsideration while maintaining learners' intellectual responsibility. Zhang et al. (2025) similarly found that purposeful teacher questioning creates opportunities to describe relationships, negotiate solutions, and extend mathematical thinking.

3.3.2 Learner-to-Learner Mathematical Discussion

Learner-to-learner mathematical discussion obtained an overall mean of 3.65, interpreted as Great Extent, but was the lowest classroom discourse dimension. Respectful responses to differing mathematical ideas obtained the highest mean of 3.95, followed by using discussion to strengthen shared conceptual understanding at 3.82. Collaborative tasks requiring learners to explain mathematical reasoning obtained 3.35, while allowing learners to question classmates' explanations received 3.38; both were interpreted as Moderate Extent. The findings indicate that classrooms provide a supportive climate for discussion, but interaction remains more developed for general participation than for sustained learner-to-learner mathematical reasoning. Jiménez-Silva et al. (2025) found that discussion-rich mathematics tasks, partner talk, visual models, and opportunities to compare representations can strengthen mathematical reasoning and language development, supporting the need for more structured peer mathematical discourse.

3.3.3 Use of Mathematical Language

The use of mathematical language was the strongest classroom discourse dimension, with an overall mean of 3.94, interpreted as Great Extent. Modeling clear mathematical explanations received the highest mean of 4.24, interpreted as Very Great Extent. Explaining mathematical vocabulary before learners used it obtained 4.13, and encouraging correct mathematical terminology received 4.08. Asking learners to explain mathematical terms in their own words received the lowest mean of 3.72. The results indicate that mathematical language instruction is particularly strong in teacher modeling and explicit vocabulary support but somewhat weaker in learner-generated language. Mathematical terminology becomes more meaningful when learners actively explain, paraphrase, discuss, and translate among verbal, symbolic, and visual forms. Jiménez-Silva et al. (2025) similarly emphasized explicit mathematical vocabulary together with partner talk, visual models, and discussion while maintaining the cognitive demand of mathematical tasks.

3.3.4 Productive Handling of Misconceptions

Productive handling of misconceptions obtained an overall mean of 3.72, interpreted as Great Extent. Using examples and non-examples to address misunderstandings received the highest mean of 3.94, followed by immediately addressing misconceptions at 3.89 and identifying common misconceptions at 3.87. Providing activities that help learners reconstruct incorrect ideas received 3.35, while asking learners to explain why a misconception produces an incorrect answer obtained 3.39; both were interpreted as Moderate Extent. The findings show that teachers effectively recognize and respond to misconceptions but rely more on teacher-directed correction than learner-led reconstruction. This parallels the pattern observed for conceptual error analysis. Mutia (2025) found that persistent mathematical difficulties require diagnostic support, discussion of errors, and opportunities to revise strategies, reinforcing the need to move from correcting misconceptions toward having learners analyze and reconstruct their own reasoning.

3.4 Relationship Between Conceptual Mathematics Teaching Practices and Learner Reasoning Support

Conceptual mathematics teaching practices and learner reasoning support demonstrated a strong, positive, and statistically significant relationship, with $r = 0.671$, $p < .001$. Teachers reporting stronger concept connection, representations, contextualization, and conceptual error analysis also tended to report stronger reasoning-oriented questioning, justification, strategy comparison, and reasoning-extending feedback. Accordingly, the null hypothesis of no significant relationship was rejected. The finding indicates that conceptual teaching and reasoning support operate as related instructional practices, although the association does not independently establish causation. The result corroborates literature linking conceptually rich instruction with reasoning through patterns, comparisons, and explanations. Ncube and Luneta (2025) further reported that concept-based instruction can strengthen conceptual understanding, critical thinking, transfer, and engagement when meaning-making is emphasized over memorized procedures.

3.5 Predictive Effects of Conceptual Mathematics Teaching Practices on Learner Reasoning Support

The final hierarchical regression model explained 55.2% of the variance in learner reasoning support, with an adjusted R^2 of .524. Concept connection and progression initially accounted for 36.1% of the variance, while multiple mathematical representations added a significant 7.2 percentage points ($p = .008$). Contextualization added 2.2 percentage points but did not produce a significant change. Adding conceptual error analysis increased explained variance by 9.7 percentage points, with F change = 12.68, $p = .001$. In the final model, conceptual error

analysis was the strongest significant predictor ($\beta = .35, p = .001$), followed by concept connection and progression ($\beta = .27, p = .010$). Multiple mathematical representations and contextualization did not retain significant unique effects. Thus, the second null hypothesis was rejected only in part. The findings distinguish descriptive strength from unique predictive importance. Although contextualization had the highest descriptive mean among conceptual teaching dimensions, it did not independently predict reasoning support after overlapping effects were controlled. Conversely, conceptual error analysis had a lower descriptive mean but the strongest unique contribution, consistent with its requirement for explanation, justification, comparison, and revision. Mutia (2025) similarly emphasized diagnostic support, explanation of errors, and strategy revision in addressing weak conceptual foundations. The significance of concept connection also aligns with literature describing mathematical understanding as cumulative and dependent on conceptual bridges and prerequisite knowledge.

3.6 Mediating Role of Classroom Mathematical Discourse Facilitation

Classroom mathematical discourse facilitation significantly mediated the relationship between conceptual mathematics teaching and learner reasoning support. Conceptual teaching significantly predicted discourse facilitation ($\beta = .70, p < .001$), while discourse significantly predicted learner reasoning support after controlling for conceptual teaching ($\beta = .47, p < .001$). The total effect was .67, whereas the direct effect declined to .34 ($p = .006$) after discourse was included. The bootstrapped indirect effect was .33, with a 95% confidence interval of [.18, .49], which excluded zero. Because the direct effect remained significant, the result indicated partial mediation, leading to rejection of the third null hypothesis. The result indicates that conceptual teaching relates to learner reasoning both directly and indirectly through mathematical discourse. Conceptual content becomes more actively connected with reasoning when learners are prompted to explain, question, discuss, justify, and revise mathematical ideas. The finding corroborates the study's proposed conceptual pathway and the reviewed synthesis connecting conceptual teaching, discourse, and reasoning. Jiménez-Silva et al. (2025) further showed that discussion-rich mathematical tasks, language scaffolds, representation, and learner explanation can work together to strengthen mathematical reasoning.

3.7 Conceptual Mathematics Teaching, Discourse, and Reasoning Enhancement Framework

The results-based framework identified concept connection and progression, multiple mathematical representations, conceptual error analysis, comparison of solution strategies, learner-to-learner mathematical discussion, productive handling of misconceptions, mathematical language, and reasoning-extending feedback as areas for sustained or strengthened practice. Concept connection remained an instructional strength and significant predictor ($\beta = .27$), while conceptual error analysis was the strongest predictor ($\beta = .35$) despite structured error-analysis activities receiving only 3.39. Comparison of solution strategies was the lowest reasoning-support domain at 3.66, learner-to-learner discussion was the lowest discourse domain at 3.65, and mathematical language was the strongest discourse domain at 3.94. The framework therefore emphasizes sustaining conceptual connections while strengthening learner-generated representations, systematic error analysis, strategy comparison and generalization, structured peer mathematical discourse, learner reconstruction of misconceptions, and feedback that initiates further reasoning. The existing strength in mathematical language can support these areas by making explanations, justification, comparison, and reasoning more explicit. The framework integrates areas of relative weakness with the significant predictors and mediating role identified in the analyses rather than relying solely on descriptive means. Davidson et al. (2025) further demonstrated the value of school-based professional learning focused on reasoning-rich lessons, analysis of learner thinking, and explanation-centered dialogue. Similarly, Weingarden and Buchbinder (2023) found that strengthening lesson design through reasoning, justification, logical connections, and discussion structures can make mathematical thinking more visible and central to instruction.

4. Conclusion & Recommendations

The study concludes that conceptual mathematics teaching, learner reasoning support, and classroom mathematical discourse facilitation are generally practiced to a great extent among Key Stage 2 teachers in Ambaguio, Nueva Vizcaya, although several learner-centered and higher-order practices require further strengthening. Conceptual teaching was strongest in contextualization and concept progression but less consistent in learner-generated representations and structured error analysis; reasoning support was strongest in explanation and justification but weaker in strategy comparison, generalization, evaluation, and transfer; and mathematical discourse was strongest in mathematical language and teacher prompting but comparatively weaker in learner-to-learner interaction and learner reconstruction of misconceptions. Conceptual mathematics teaching was significantly and positively related to learner reasoning support, while conceptual error analysis and concept connection and progression emerged as significant predictors, with conceptual error analysis providing the strongest unique contribution. Classroom mathematical discourse also partially mediated the relationship between conceptual teaching and reasoning support, indicating that conceptual instruction supports reasoning both directly and through richer mathematical discourse. These findings support an integrated instructional enhancement framework that maintains existing strengths while strengthening the identified areas of need.

Key Stage 2 mathematics instruction should further strengthen explicit conceptual connections, learner-generated mathematical representations, higher-order reasoning, structured conceptual error analysis, peer mathematical discourse, misconception reconstruction, and reasoning-extending feedback while sustaining strengths in contextualization, mathematical language, and concept progression. Teachers may provide greater opportunities for learners to compare and evaluate strategies, generalize and transfer mathematical ideas, analyze and revise errors, question and build on peers' explanations, and justify their mathematical thinking. School heads, master teachers, and mathematics coordinators may support these practices through professional development, coaching, lesson study, collaborative planning, classroom observation, and Learning Action Cell activities that integrate conceptual teaching with purposeful mathematical discourse. The proposed instructional enhancement framework may also be implemented, monitored, and refined through school-based mathematics improvement initiatives.

The study was limited to Key Stage 2 teachers in Ambaguio, Nueva Vizcaya, which restricts the extent to which the observed patterns can be applied to broader educational settings. The recommendations for future research indicate limitations associated with sample coverage and the absence of additional evidence from classroom observations, learner achievement measures, interviews, mathematical reasoning performance tasks, and longitudinal data. Consequently, the findings primarily reflect the patterns observed among the participating teachers and do not establish whether these patterns are similarly evident in actual learner outcomes or remain consistent across other districts, divisions, grade levels, and educational contexts. The proposed instructional enhancement framework also requires further testing through intervention or quasi-experimental studies.

5. Compliance with ethical standards

The study adhered to ethical standards throughout the research process. Appropriate permissions were obtained from the concerned education offices and school authorities, and participation was voluntary and based on informed consent. Respondents were informed of the study's purpose, the nature of their participation, their right to refuse or withdraw without negative consequences, and the confidentiality of their responses. No sensitive personal information or profile variables were collected, and the identities of respondents and schools were protected. Data were used solely for academic and research purposes, securely handled and reported in summary form to prevent individual identification, and presented accurately and honestly without alteration, fabrication, or misrepresentation.

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