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Mathematical Flexibility Observed in the Various Stages of Compression for Foundations in Algebra Problems

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Abstract

This study examined mathematical flexibility among high school students in solving Foundations in Algebra problems across procedural and proceptual stages of mathematical compression. A convergent mixed-methods design was employed involving 60 students, comprising 31 Regular and 29 College Prep students, selected through convenience sampling at Spring Valley High School, Columbia, South Carolina, United States. Data were collected using a researcher-developed assessment involving exponential expressions and follow-up interviews with 22 students. Descriptive statistics, the Mann–Whitney test, and qualitative content analysis were utilized. Results revealed higher accuracy in non-proceptual problems (86.7%) than in proceptual problems (43.4%). Strategy generation reached 49.3%, while practical flexibility was lower at 32.3%, indicating limited spontaneous application of innovative strategies. College Prep students demonstrated significantly higher strategy generation ($p = .03$) and strategy evaluation ($p = .01$) than Regular students, although practical flexibility did not differ significantly ($p = .23$). The findings indicate that mathematical flexibility supports conceptual reasoning and successful algebraic problem-solving, although procedural errors may affect accuracy. Mathematics teachers are encouraged to integrate alternative solution strategies, conceptual explanations, and proceptual tasks into instruction. The single-school convenience sample and focus on exponential expressions limit generalizability. The study contributes evidence for strengthening adaptive mathematics instruction and supports Sustainable Development Goal 4 (Quality Education) by promoting conceptual understanding, flexible reasoning, and sustainable improvements in mathematical learning.

Keywords: Algebraic Problem-Solving; Conceptual Understanding; Mathematical Flexibility; Mixed-Methods Research; Practical Flexibility; Proceptual Thinking; Strategy Generation

1. Introduction

Mathematical flexibility enables students to recognize mathematical structures, generate alternative solutions, and select strategies appropriate to the demands of a problem. Recent syntheses distinguish knowing multiple methods from choosing and executing an effective method, emphasizing that flexible reasoning is a central outcome of mathematics learning (Hong et al., 2023; Verschaffel, 2024). Wang and Star (2023) further distinguish structure-informed flexibility from preference for standard algorithms. These distinctions are particularly important in algebra, where students may successfully perform routine calculations yet struggle to interpret symbolic expressions conceptually.

Conceptual knowledge concerns the relationships and principles underlying mathematical ideas, whereas procedural knowledge supports the accurate application of operations and algorithms. Both are needed for meaningful algebraic reasoning (Hakim & Yasmadi, 2021; Hurrell, 2021). Research with algebra learners has demonstrated the value of examining the relationship between conceptual and procedural understanding rather than treating the two as competing forms of knowledge (Manandhar et al., 2022; Cabuquin & Abocejo, 2024). Evidence from Grade 9 algebra instruction also points to the importance of attending to conceptual and procedural approaches for students with differing levels of mathematical difficulty (Lee, 2025).

An important distinction is that students may possess knowledge of alternative strategies without applying those strategies spontaneously. Garcia Coppersmith and Star (2022) found that strategy choice, accuracy, and problem type interact in complex ways, indicating that flexible strategy use does not always guarantee a correct answer. Recent research has likewise emphasized the value of examining students' strategy selection and the reasoning behind their choices (Hong et al., 2023; Wang & Star, 2023). Such evidence supports assessing strategy generation, evaluation, potential flexibility, and practical flexibility as related but distinct aspects of mathematical problem-solving.

Instructional research indicates that opportunities to compare and discuss multiple methods can strengthen algebra learning and procedural flexibility. Durkin et al. (2023) reported improved algebra learning when classrooms systematically compared alternative solutions, while Hämmerle (2023) showed that classroom discussions comparing methods provided more opportunities to plan equation-solving strategies. Related work has investigated how educational technologies may support conceptual knowledge, procedural knowledge, and procedural flexibility (Chan et al., 2023). Other studies emphasize deliberate problem-solving representations and procedures that help learners reason beyond rote execution (Schulz, 2024; Chinofunga et al., 2025). Together, these findings support instruction that encourages students to explain, compare, and justify different algebraic approaches (Santos-Trigo, 2024; Asilo-Ebisa & Lomibao, 2024).

The theoretical framework of this investigation is proceptual thinking, which describes how mathematical symbols can represent both a process and a conceptual object. Gray and Tall (1991, 1992) established the foundational distinction between procedural activity and flexible proceptual reasoning. The assessment also draws on Xu et al. (2017), who distinguished potential flexibility from practical flexibility in equation solving. These foundational sources are retained because they define the central constructs and assessment logic rather than merely providing historical background.

Against this background, the study examined mathematical flexibility among students at Spring Valley High School in Columbia, South Carolina, United States, using Foundations in Algebra tasks involving exponents. Specifically, it investigated accuracy in problems requiring and not requiring proceptual thinking, strategy generation and evaluation, differences between Regular and College Prep students, and the relationship between flexible strategies and success in proceptual tasks. The study supports Sustainable Development Goal 4 (Quality Education) by informing mathematics instruction that integrates conceptual understanding, procedural accuracy, and adaptive problem-solving (Verschaffel, 2024; Santos-Trigo, 2024).

2. Material and methods

2.1 Research Design

The study utilized a convergent mixed-methods research design, integrating quantitative descriptive analysis and qualitative examination of students' mathematical problem-solving strategies. Following Creswell (2012), this design involves collecting quantitative and qualitative data concurrently, analyzing them separately, and comparing the findings to develop a comprehensive understanding of the research problem. The quantitative component assessed students' accuracy in solving FIA problems that did not require proceptual thinking (Problem Type 1) and those requiring proceptual thinking (Problem Type 2). It also examined strategy generation, strategy evaluation, potential flexibility, and practical flexibility to determine students' ability to produce, recognize, and apply alternative mathematical solutions. The qualitative component analyzed the strategies employed by Regular and College Prep students, focusing on how they developed and explained their mathematical approaches. Thematic analysis was applied to identify common patterns in their responses. Quantitative and qualitative findings were subsequently compared and integrated to explain how students demonstrated mathematical flexibility across procedural and proceptual problem-solving tasks. This approach enabled the investigation to examine both measurable differences in mathematical performance and the reasoning underlying students' selection of strategies.

2.2 Locale of the Study

The study was conducted at Spring Valley High School in Columbia, South Carolina, United States of America. The institution is a public magnet school offering specialized academic programs, including the Discovery Program, a four-year Advanced Placement and Honors program emphasizing science, technology, engineering,

and mathematics (STEM) research, and the Explorations Program, a four-year College Prep program designed for students interested in science, mathematics, and technology. The school was selected because it provided an appropriate setting for examining mathematical flexibility among students with different levels of algebraic preparation. The researcher was also employed at the institution as a mathematics teacher for Grades 9 and 12, making the research setting accessible for administering assessments and conducting interviews.

2.3 Respondents of the Study

The respondents consisted of 60 high school students who had completed Foundations in Algebra or Algebra 1, or were enrolled in Intermediate Algebra. These courses provided the mathematical background necessary for assessing students' ability to solve algebraic problems using procedural and proceptual approaches. The participants were classified into two groups according to their mathematics coursework. The Regular group consisted of 31 students who had completed Foundations in Algebra or were taking Intermediate Algebra, while the College Prep group comprised 29 students who had completed Algebra 1. Foundations in Algebra and Intermediate Algebra form a two-course sequence intended to provide additional mathematical support to students preparing for Algebra 1. The distinction between the two groups enabled comparisons of mathematical flexibility according to their algebraic preparation.

2.4 Sample and Sampling Procedure

The study employed convenience sampling to select 60 students who voluntarily agreed to participate and provided informed consent. Participants were selected based on their accessibility, willingness to participate, and relevant mathematics coursework. The sample was organized into two groups comprising 31 Regular students and 29 College Prep students. This classification facilitated comparisons of problem-solving accuracy, strategy generation, strategy evaluation, and mathematical flexibility between students with different levels of algebraic preparation.

2.5 Research Instrument

The study utilized a researcher-developed questionnaire designed to assess mathematical flexibility in Foundations in Algebra, particularly in solving problems involving exponents. The instrument consisted of 10 questions divided into two categories: five Problem Type 1 questions that did not require proceptual thinking and five Problem Type 2 questions that required proceptual thinking. The questionnaire was examined, validated, and approved by the research adviser. The assessment examined students' accuracy, strategy generation, strategy evaluation, potential flexibility, and practical flexibility. Problem Type 1 assessed students' ability to solve mathematical expressions through standard computational procedures or innovative approaches, while Problem Type 2 required deeper conceptual understanding and flexible mathematical reasoning. The questions involved simplifying exponential expressions, applying exponent rules, rewriting expressions in condensed forms, and comparing algebraic quantities. For example, students could simplify an expression involving several powers of the same base through direct computation or by factoring common exponential terms. Other problems required students to apply exponent properties and explain relationships between algebraic expressions. These tasks enabled the identification of standard and innovative solution strategies and provided evidence of students' ability to interpret mathematical expressions both procedurally and conceptually. The instrument also supported comparisons between Regular and College Prep students. Follow-up interview questions assessed whether participants could recognize correct solutions, identify innovative approaches, and explain their mathematical reasoning. The evaluation of mathematical flexibility was guided by the assessment approach of Xu et al. (2017).

2.6 Data Gathering Procedure

Data collection commenced after obtaining permission from the Director of Accountability, Research and Evaluation of Richland School District Two and the principal of Spring Valley High School. Approval was requested in July 2021, followed by the administration of the researcher-developed questionnaire in October 2021. The assessment procedures were adapted from the Four-Phase Flexibility Assessment developed by Xu et al. (2017). The assessment and interviews were conducted in three phases, followed by a fourth phase involving analysis of the collected responses. During the first phase, participants answered five Problem Type 1 and five Problem Type 2 questions involving exponents within 30 minutes. Students were instructed to solve the problems accurately and efficiently using their preferred strategies. After a ten-minute break, the second phase required participants to solve the same problems using alternative approaches within 60 minutes. Students were not permitted to modify their initial solutions, allowing the researcher to distinguish their first strategies from subsequently generated alternatives. During the third phase, students participated in brief interviews lasting approximately 2–10 minutes. They were asked to identify which of their strategies were correct and, when multiple correct solutions were available, determine which approach was more innovative and explain their reasoning. The interviews were audio-recorded to preserve the participants' explanations and support subsequent qualitative analysis. The fourth phase was conducted in November 2021 and involved organizing, evaluating, and comparing students' mathematical solutions and interview responses. The researcher examined differences between Regular and College Prep students and identified patterns in strategy generation, strategy evaluation, and

mathematical flexibility. The combined assessment and interview procedures provided quantitative measures of performance and qualitative explanations of students' problem-solving approaches.

2.7 Data Analysis Techniques

The collected data were analyzed using descriptive statistics, the Mann–Whitney test, and qualitative content analysis. The procedures for evaluating mathematical flexibility were adapted from Xu et al. (2017), particularly the measures of potential and practical flexibility in equation solving. Students' problem-solving accuracy was determined by assigning one point for each correct answer and zero points for incorrect answers. Each problem category had a maximum accuracy score of five points. Frequencies, means, and percentages were calculated to describe students' performance in solving problems that required and did not require proceptual thinking. Mathematical flexibility was evaluated through strategy generation, strategy evaluation, potential flexibility, and practical flexibility. Strategy generation was scored one point when students demonstrated knowledge of both standard and innovative solutions and zero otherwise. Strategy evaluation was assessed by determining whether students correctly identified an innovative strategy among their generated solutions. Potential flexibility was awarded one point when students successfully demonstrated both strategy generation and strategy evaluation for a given problem. Practical flexibility was assessed by examining students' initial solutions, with one point assigned when an innovative strategy was used during the first attempt. These measures were also examined in relation to correct and incorrect responses to determine how mathematical flexibility was associated with problem-solving success. Differences between Regular and College Prep students were examined using the Mann–Whitney test, which compared their mathematical flexibility scores. Comparisons of accuracy, strategy generation, and practical flexibility involved 60 students (29 College Prep and 31 Regular), whereas comparisons of strategy evaluation and potential flexibility involved 22 interviewed students (14 College Prep and 8 Regular). Statistical significance was evaluated at the 0.05 level. Qualitative content analysis was also employed to identify common themes in students' written solutions and interview responses, particularly regarding their ability to generate, recognize, and explain alternative mathematical strategies. Finally, the researcher compared the strategies of students who successfully solved both procedural and proceptual problems with those who solved only procedural problems. This analysis examined how flexible mathematical strategies related to successful proceptual thinking. The integration of descriptive statistical findings and qualitative interpretations provided a comprehensive assessment of students' mathematical flexibility across different stages of mathematical compression.

3. Results and Discussion

3.1 Proceptual Thinking in Solving Foundations in Algebra Problems

3.1.1 Performance in Problems Requiring and Not Requiring Proceptual Thinking

The findings revealed that students performed better in problems that did not require proceptual thinking (Problem Type 1) than in those requiring proceptual thinking (Problem Type 2). College Prep and Regular students achieved comparable performance in Problem Type 1, obtaining 83.9% and 89.7%, respectively, with a combined performance of 86.7%. However, their combined performance decreased to 43.4% in Problem Type 2. College Prep students demonstrated higher accuracy in proceptual problems than Regular students, achieving 53.0% compared with 33.6%. These results indicate that both groups demonstrated stronger proficiency in routine algebraic operations than in problems requiring deeper conceptual reasoning. The findings suggest that students possessed adequate foundational algebraic knowledge but experienced difficulties when required to apply flexible mathematical thinking. Although both groups performed well in procedural tasks, their lower performance in proceptual problems indicates limitations in applying mathematical concepts beyond familiar computational procedures. College Prep students demonstrated relatively stronger proceptual reasoning, although their performance also declined compared with routine problems. These findings highlight the importance of strengthening students' ability to connect mathematical procedures with conceptual understanding, particularly in preparation for the End-of-Course Program Examination required by the South Carolina Department of Education.

3.2 Students' Mathematical Flexibility in Solving Foundations in Algebra Problems

3.2.1 Strategy Generation and Practical Flexibility

The findings showed that students demonstrated a strategy generation rate of 49.3% in solving non-proceptual problems, indicating their ability to produce both standard and innovative solutions. However, practical flexibility was lower at 32.3%, suggesting that students did not consistently apply innovative strategies during their initial attempts. Strategy generation was highest in the first and fourth problems, at 63.3% and 76.7%, respectively, while practical flexibility for these problems was 16.7% and 45.0%. In the second problem, both strategy generation and practical flexibility were 0%, although 86.7% of students obtained correct answers. Of the 300 student responses, 153 demonstrated both standard and innovative strategies, resulting in 148 correct and five incorrect solutions. Thus, strategy generation was demonstrated in 51.0% of all responses when both correct and incorrect solutions were considered. These findings indicate that students were generally capable of generating alternative mathematical strategies but preferred familiar procedures when initially solving problems. Their tendency to evaluate exponential expressions directly rather than apply factoring suggests that procedural approaches remained more convenient, even when innovative solutions were available. Nevertheless, the high proportion of

correct answers among students who demonstrated multiple strategies suggests that knowledge of alternative approaches supported mathematical accuracy. The findings also highlight the distinction between possessing flexible mathematical knowledge and applying it spontaneously during problem-solving.

3.2.2 Strategy Evaluation and Potential Flexibility

Among the 22 students interviewed, strategy evaluation reached 57.3%, indicating that more than half could correctly identify innovative solutions. Strategy evaluation was particularly high in the first and fourth problems, at 72.7% and 81.8%, respectively. Potential flexibility was recorded at 45.5%, demonstrating that some students could both generate alternative solutions and identify innovative strategies. Although practical flexibility was lower at 32.3% in the overall assessment, students demonstrated the ability to recognize innovative approaches even when these were not used during their initial attempts. The findings suggest that students possessed mathematical knowledge extending beyond their preferred computational procedures. Their ability to identify innovative strategies indicates conceptual awareness, although this knowledge was not consistently reflected in their initial problem-solving approaches. The difference between potential and practical flexibility emphasizes that recognizing an efficient mathematical strategy does not necessarily result in its immediate application. Consequently, mathematical instruction may benefit from providing opportunities for students to practice selecting and applying alternative approaches independently.

3.3 Comparison of Mathematical Flexibility Between Regular and College Prep Students

3.3.1 Differences in Strategy Generation and Practical Flexibility

College Prep students demonstrated higher strategy generation than Regular students, obtaining 55.2% compared with 43.9%. College Prep students also showed greater practical flexibility in the first, third, and fourth non-proceptual problems. However, Regular students demonstrated more innovative solutions in the fifth problem, particularly in applying their foundational understanding of exponential expressions. The Mann–Whitney test revealed a statistically significant difference in strategy generation between the two groups ($z=-2.19$, $p=0.03$), favoring College Prep students. In contrast, practical flexibility did not differ significantly between the groups ($p=0.23$). These findings indicate that College Prep students possessed a broader repertoire of mathematical strategies, enabling them to generate alternative solutions more effectively. However, their greater knowledge of innovative approaches did not consistently translate into their immediate application. Regular students also demonstrated flexibility in selected problems, particularly those involving familiar algebraic concepts. The results suggest that differences in mathematical preparation may influence students' ability to generate alternative strategies, although both groups continued to rely on conventional procedures when solving routine problems.

3.3.2 Differences in Strategy Evaluation and Potential Flexibility

College Prep students achieved higher strategy evaluation scores than Regular students, obtaining 62.9% and 47.5%, respectively. Similarly, potential flexibility was higher among College Prep students at 52.9%, compared with 32.5% among Regular students. The Mann–Whitney test identified a statistically significant difference in strategy evaluation ($z=2.46$, $p=0.01$), favoring College Prep students. Overall, College Prep students demonstrated significantly stronger strategy generation ($p=0.03$) and strategy evaluation ($p=0.01$), although the difference in practical flexibility remained statistically insignificant ($p=0.23$). The results suggest that College Prep students were more capable of recognizing and evaluating innovative mathematical strategies, reflecting stronger potential flexibility. Their greater exposure to algebraic concepts may have contributed to their ability to identify alternative approaches. Nevertheless, Regular students also demonstrated mathematical competence, with 14 of the 31 students obtaining perfect scores in non-proceptual problems and four achieving four correct answers out of five in proceptual problems. These findings indicate that students receiving additional algebraic support can also demonstrate mathematical flexibility, although differences in strategy knowledge and evaluation remain evident.

3.4 Comparison of Mathematical Flexibility in Proceptual and Non-Proceptual Problems

3.4.1 Strategy Generation Across Problem Types

The analysis of students' written solutions revealed differences in strategy generation between proceptual and non-proceptual problems. In the first non-proceptual problem, a College Prep participant (P11) successfully demonstrated both standard and innovative strategies, whereas a Regular participant (P10) produced several solutions that were primarily variations of conventional procedures. Students generally preferred evaluating exponential expressions directly rather than applying exponent rules or factoring. Practical flexibility for this problem was 24.1% among College Prep students and 9.7% among Regular students. In contrast, the solutions of P3 from College Prep and P12 from Regular demonstrated that successful completion of a proceptual problem required integrating additional algebraic principles, including the cancellation and manipulation of exponential expressions. These findings suggest that non-proceptual problems provide opportunities for students to explore multiple solutions, although learners frequently prefer familiar computational methods. Proceptual problems, however, require students to move beyond routine procedures and apply mathematical concepts more flexibly. The differences observed in students' solutions indicate that mathematical flexibility involves not only producing alternative approaches but also recognizing when conventional strategies are insufficient to solve a particular problem.

3.4.2 Strategy Evaluation and Potential Flexibility

The examination of students' solutions and interview responses demonstrated differences in strategy evaluation and potential flexibility. In the fourth non-proceptual problem, both P4 from College Prep and P38 from Regular correctly identified innovative strategies involving the conversion of exponential expressions into equivalent bases. However, P4 applied the innovative approach during the initial attempt, whereas P38 generated the strategy during the second phase. In another problem requiring proceptual thinking, P31 from College Prep successfully applied additional algebraic concepts, including common monomial factoring, while P26 from Regular relied on standard procedures and obtained an incorrect answer. These findings demonstrate that students may recognize innovative mathematical strategies without necessarily applying them immediately. The distinction between the two participants in the non-proceptual problem illustrates how practical flexibility depends on the spontaneous application of efficient approaches, while potential flexibility reflects the ability to generate and recognize alternative strategies. The successful solution of the College Prep participant in the proceptual problem further indicates the importance of connecting foundational algebraic procedures with broader mathematical concepts.

3.4.3 Practical Flexibility in Mathematical Problem-Solving

The findings showed that College Prep students demonstrated practical flexibility of 37.9%. Examination of selected solutions revealed that P30 successfully explained algebraic relationships in a non-proceptual problem using an innovative approach, while P31 demonstrated flexible reasoning when solving a proceptual problem. These students applied mathematical concepts beyond conventional computational procedures, showing their ability to select appropriate strategies and explain mathematical relationships. In contrast, other participants frequently used standard methods even when alternative solutions were available. The findings indicate that practical flexibility is demonstrated when learners independently apply innovative strategies to solve mathematical problems. Non-proceptual problems allow students to choose between standard and alternative procedures, whereas proceptual problems require greater conceptual manipulation to obtain correct solutions. The observed differences suggest that students can develop flexible mathematical thinking through exposure to both types of problems, particularly when encouraged to examine mathematical relationships beyond routine computation.

3.5 Relationship Between Flexible Strategies and Success in Solving Proceptual Problems

3.5.1 Application of Flexible Strategies in Algebraic Problem-Solving

The analysis of students' solutions demonstrated how strategies developed in non-proceptual problems were applied to more complex proceptual tasks. Participant P4 successfully solved a proceptual problem by converting exponential expressions into equivalent bases, applying exponent rules, and factoring the greatest common factor. These strategies were also evident in the participant's earlier non-proceptual solutions. Similarly, P2 successfully solved another proceptual problem by applying factoring and the quotient rule, demonstrating the transfer of strategies previously used in a simpler problem. Participant P4 also applied expanded exponential expressions in another proceptual task, reflecting the ability to manipulate mathematical expressions beyond routine simplification. These findings suggest that flexible strategies developed through foundational algebraic problems can support successful performance in tasks requiring proceptual thinking. Students who understand relationships among mathematical expressions can apply previously learned concepts to unfamiliar problems rather than depending exclusively on memorized procedures. This interpretation is consistent with Durkin et al. (2023), who found that comparing multiple algebraic strategies can support procedural flexibility. The successful application of alternative strategies highlights the importance of connecting procedural competence with conceptual understanding.

3.5.2 Mathematical Flexibility and Problem-Solving Errors

The findings also revealed that possessing flexible strategies did not always guarantee correct answers. Participant P11 demonstrated knowledge of exponent rules and appropriate simplification procedures but obtained an incorrect answer because of an error in manipulating exponential expressions. Similarly, P52 demonstrated flexible reasoning by converting numerical expressions into equivalent bases and applying exponent rules, although an incorrect conversion resulted in an inaccurate final answer. In contrast, P9 relied primarily on conventional procedures and demonstrated difficulty solving a proceptual problem requiring more flexible mathematical manipulation. These findings indicate that mathematical flexibility and computational accuracy are related but distinct aspects of problem-solving performance. Students may demonstrate appropriate mathematical reasoning and alternative strategies while committing procedural errors that affect their final answers. Conversely, reliance on conventional approaches may restrict students' ability to solve unfamiliar problems requiring conceptual manipulation. The findings emphasize the importance of developing both flexible reasoning and procedural accuracy to improve performance in algebraic problem-solving.

3.5.3 Contribution of Flexible Strategies to Proceptual Thinking

The overall findings demonstrated that students who successfully applied alternative strategies in non-proceptual problems were better positioned to address mathematical tasks requiring proceptual thinking. Successful

participants demonstrated the ability to recognize equivalent expressions, manipulate exponential relationships, apply factoring techniques, and integrate different algebraic concepts. Students who relied exclusively on conventional procedures encountered greater difficulties when solving problems that required deeper mathematical reasoning. Nevertheless, some participants demonstrated flexible strategies without obtaining correct answers because of computational errors. These findings support the conclusion that mathematical flexibility contributes to successful proceptual thinking by enabling students to move beyond memorized procedures and apply mathematical concepts across different problem situations. This interpretation is consistent with Hong et al. (2023) and Verschaffel (2024), who describe flexibility as an important dimension of mathematical problem-solving and strategy selection. Overall, the results emphasize the importance of integrating procedural knowledge, conceptual understanding, and innovative problem-solving strategies to strengthen students' mathematical flexibility and improve their performance in Foundations in Algebra.

4. Conclusion & Recommendations

The study concluded that mathematical flexibility plays an important role in students' ability to solve Foundations in Algebra problems across procedural and proceptual stages of mathematical compression. Students demonstrated stronger performance in problems that did not require proceptual thinking, indicating adequate foundational algebraic skills and the ability to generate alternative strategies. However, problems requiring proceptual thinking presented greater challenges, particularly when students needed to apply conceptual understanding beyond routine procedures. College Prep students generally exhibited higher mathematical flexibility than Regular students, although knowledge of multiple strategies did not consistently result in their immediate application. Difficulties in interpreting negative exponents as mathematical concepts further highlighted limitations in conceptual understanding. Overall, the findings indicated that mathematical flexibility was associated with successful proceptual problem-solving. The study contributes to mathematics education by emphasizing the importance of integrating procedural competence, conceptual understanding, and innovative strategies to support more accurate and efficient algebraic problem-solving.

Based on the findings, mathematics teachers are encouraged to incorporate both proceptual and non-proceptual problems into classroom instruction to strengthen students' mathematical flexibility. Non-proceptual activities should provide opportunities for learners to generate, compare, and evaluate multiple solution strategies, while proceptual problems should encourage deeper conceptual reasoning and the application of innovative approaches. Greater instructional emphasis should be placed on exponential expressions, particularly negative exponents, exponent rules, and expanded forms, to address difficulties in understanding mathematical processes and concepts. Teachers should also encourage students to explain their reasoning and recognize efficient alternative solutions rather than relying exclusively on familiar procedures. Educational institutions may conduct professional development sessions or mini-lectures focusing on mathematical flexibility and proceptual thinking to strengthen instructional practices. Future researchers are encouraged to examine mathematical flexibility across other algebraic topics and mathematical domains, using probability sampling to improve sample representativeness and extend the applicability of the findings.

The study was limited to 60 students from Spring Valley High School in Columbia, South Carolina, United States, comprising 31 Regular and 29 College Prep students. Convenience sampling restricted the representativeness of the participants, while only 22 students participated in follow-up interviews. The investigation focused exclusively on foundational algebraic problems involving exponents, limiting its applicability to other mathematical topics and levels of education. Furthermore, the assessment examined students' mathematical flexibility within a specific data collection period rather than across extended learning experiences. Future studies may employ probability sampling, include participants from multiple educational institutions, and investigate additional mathematical topics to broaden the applicability of the findings.

5. Compliance with ethical standards

The author affirms that no conflicts of interest exist in relation to this study. The research involved 60 students from Spring Valley High School in Columbia, South Carolina, USA, with institutional permission obtained from the Richland School District Two Director of Accountability, Research and Evaluation and the school principal. Participation was voluntary, informed consent was obtained, and participants were informed of their right to withdraw without prejudice; data were collected through mathematical assessments and audio-recorded interviews. Confidentiality safeguards included assigning participant codes, excluding identifying information from reports, restricting access to research records, and securely storing collected data, with participants and the school administration acknowledged for their cooperation and support.

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